xPand. Documentation

This is the doc file xPandDoc.nb of version 0.4.0 of xAct/Pand`. Last update on 8 February 2013.

http://www2.iap.fr/users/pitrou/xpand.htm

Preamble

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Intro

xAct`xPand` is a Mathematica Package which provides tools to compute formally the cosmological perturbations around an homogeneous background.

It relies on a 3+1 splitting of the background spacetimes, and uses the perturbative tools developed in xAct`xPert` which itself is part of the xAct` suite.

More details about the algorithm can be found on a dedicated publication.

Loading the package

Loading the package is straightforward once it has been appropriately installed:

You just have to evaluate: '<< xAct`xPand.m;'

This loads all the definitions, but does not start any computation.

We can also check how much memory space it takes to store all the definitions, and the time it takes to load them (less than 1 second!):

```
In[1]:= MemBefore = MemoryInUse[];
        << xAct/xPand.m; // Timing // First

          ------------------------------------------------------------

          Package xAct`xPerm`  version 1.2.0, {2013, 1, 27}

          CopyRight (C) 2003-2013, Jose M.
            Martin-Garcia, under the General Public License.

          Connecting to external mac executable...

          Connection established.

          ------------------------------------------------------------

          Package xAct`xTensor`  version 1.0.5, {2013, 1, 27}

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            Martin-Garcia, under the General Public License.

          ------------------------------------------------------------

          Package xAct`xPert`  version 1.0.3, {2013, 1, 27}

          CopyRight (C) 2005-2013, David Brizuela, Jose M. Martin-Garcia
            and Guillermo A. Mena Marugan, under the General Public License.

          ** Variable $PrePrint assigned value ScreenDollarIndices

          ** Variable $CovDFormat changed from Prefix to Postfix

          ** Option AllowUpperDerivatives of ContractMetric changed from False to True

          ** Option MetricOn of MakeRule changed from None to All

          ** Option ContractMetrics of MakeRule changed from False to True

          ** DefInertHead: Defining inert head Perturbation.

          ------------------------------------------------------------

          Package xAct`xPand`  version 0.4.0, {2013, 2, 8}

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            Roy and Obinna Umeh under the General Public License.

          ------------------------------------------------------------

          These packages come with ABSOLUTELY NO WARRANTY; for details type
            Disclaimer[]. This is free software, and you are welcome to redistribute
            it under certain conditions. See the General Public License for details.

          ------------------------------------------------------------

Out[2]= 0.988017

In[3]:= MemoryInUse[] - MemBefore

Out[3]= 26 728 360
```

GPL

```
In[4]:= Disclaimer[]

        These are points 11 and 12 of the General Public License:

        BECAUSE THE PROGRAM IS LICENSED FREE OF CHARGE, THERE IS NO WARRANTY
          FOR THE PROGRAM, TO THE EXTENT PERMITTED BY APPLICABLE LAW. EXCEPT
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          (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA BEING RENDERED
          INACCURATE OR LOSSES SUSTAINED BY YOU OR THIRD PARTIES OR A FAILURE OF
          THE PROGRAM TO OPERATE WITH ANY OTHER PROGRAMS), EVEN IF SUCH HOLDER
          OR OTHER PARTY HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES.
```

Useful tools

```
In[5]:= org[expr_] := ScreenDollarIndices@
          NoScalar@Collect[ContractMetric[expr], $PerturbationParameter, ToCanonical]
        collect[expr_] := ScreenDollarIndices@
          NoScalar@Collect[expr, $PerturbationParameter, ContractMetric]
```

Overview

Tools

■ Background slicing of the manifold

We must first define a manifold, choose the dimension of the space-time and the set of indices to work with. The user can also choose any kind of indices.

Then we define a metric and assign a signature to it. The operational and formating notation for the covariant derivative associated with the metric are also chosen.

```
In[7]:= DefManifold[M, 4, {α, β, γ, μ, ν, λ, σ}]
        DefMetric[-1, g[-α, -β], CD, {";", "∇̄"}, PrintAs → "ḡ"];

    ** DefManifold: Defining manifold M.
    ** DefVBundle: Defining vbundle TangentM.
    ** DefTensor: Defining symmetric metric tensor g[-α, -β].
    ** DefTensor: Defining antisymmetric tensor epsilong[-α, -β, -γ, -λ].
    ** DefTensor: Defining tensor Tetrag[-α, -β, -γ, -λ].
    ** DefTensor: Defining tensor Tetrag†[-α, -β, -γ, -λ].
    ** DefCovD: Defining covariant derivative CD[-α].
    ** DefTensor: Defining vanishing torsion tensor TorsionCD[α, -β, -γ].
    ** DefTensor: Defining symmetric Christoffel tensor ChristoffelCD[α, -β, -γ].
    ** DefTensor: Defining Riemann tensor RiemannCD[-α, -β, -γ, -λ].
    ** DefTensor: Defining symmetric Ricci tensor RicciCD[-α, -β].
    ** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
    ** DefTensor: Defining Ricci scalar RicciScalarCD[].
    ** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.
    ** DefTensor: Defining symmetric Einstein tensor EinsteinCD[-α, -β].
    ** DefTensor: Defining Weyl tensor WeylCD[-α, -β, -γ, -λ].
    ** DefTensor: Defining symmetric TFRicci tensor TFRicciCD[-α, -β].
    ** DefTensor: Defining Kretschmann scalar KretschmannCD[].
    ** DefCovD:  Computing RiemannToWeylRules for dim 4
    ** DefCovD:  Computing RicciToTFRicci for dim 4
    ** DefCovD:  Computing RicciToEinsteinRules for dim 4
    ** DefTensor: Defining weight +2 density Detg[]. Determinant.
```

We have a general function called 'SetSlicing' which defines an induced metric h on the hypersurfaces and a vector n normal to them. The conformal metric is also defined here.

Many other relations that are required for the (N-1)+1 splitting are implemented. One key aspect of SetSlicing is that we choose the type of space-time to work with when calling the function.

At the moment, the possible space – time options include

Minkowski
FLCurved
 (for postively or negatively curved Friedmann – Lema ître – Robertson – Walker (FLRW) space – times)
FLFlat (for flat FLRW space – times)
BianchiB (for general Bianchi space – times with the most general constants of structure)
BianchiA (for Type A Bianchi space – times, where the ' vector' a_μ vanishes)
BianchiI (for Type I Bianchi space – times)

In[9]:= **? SetSlicing**

SetSlicing[g,n,nNorm,h,cd,{cdPost,cdPre},SpaceTimeType].

Given a background manifold of dimension 'N' and an ambient metric 'g' on that manifold, this function performs a (N−1)+1 slicing.

More precisely, it builds the induced metric 'h' on the hypersurfaces, the associated covariant derivative 'cd', and the vector 'n' normal to these hypersurfaces. It also sets the type 'SpaceTimeType' for the hypersurfaces. The latter argument can take a value among {Anisotropic, BianchiB, BianchiA, BianchiI, FLCurved, FLFlat, Minkowski}. The norm 'nNorm' of the vector 'n' is set by default to −1 if omitted.

In[10]:= **SetSlicing[g, n, h, cd, {"|", "D̄"}, "FLFlat"]**

```
** DefTensor: Defining tensor n[σ$1474].
** DefTensor: Defining symmetric metric tensor h[-σ$1474, -σ$1475].
** DefTensor: Defining antisymmetric tensor epsilonh[-α, -β, -γ].
** DefTensor: Defining tensor Tetrah[-α, -β, -γ, -λ].
** DefTensor: Defining tensor Tetrah†[-α, -β, -γ, -λ].
** DefCovD: Defining covariant derivative cd[-σ$1474].
** DefTensor: Defining vanishing torsion tensor Torsioncd[α, -β, -γ].
** DefTensor: Defining symmetric Christoffel tensor Christoffelcd[α, -β, -γ].
** DefTensor: Defining Riemann tensor Riemanncd[-α, -β, -γ, -λ].
** DefTensor: Defining symmetric Ricci tensor Riccicd[-α, -β].
** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
** DefTensor: Defining Ricci scalar RicciScalarcd[].
** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.
** DefTensor: Defining symmetric Einstein tensor Einsteincd[-α, -β].
** DefTensor: Defining Weyl tensor Weylcd[-α, -β, -γ, -λ].
** DefTensor: Defining symmetric TFRicci tensor TFRiccicd[-α, -β].
** DefTensor: Defining Kretschmann scalar Kretschmanncd[].
** DefCovD:  Computing RiemannToWeylRules for dim 4
** DefCovD:  Computing RicciToTFRicci for dim 4
** DefCovD:  Computing RicciToEinsteinRules for dim 4
** DefTensor: Defining weight +2 density Deth[]. Determinant.
```

```
** DefTensor: Defining extrinsic curvature tensor
 ExtrinsicKh[α, β]. Associated to vector n

** DefTensor: Defining acceleration vector
 Accelerationn[α]. Associated to vector n

** DefInertHead: Defining projector inert-head Projectorh.

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for h.

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for h.

   Rules {1, 2} have been declared as UpValues for n.

   Rules {1, 2, 3, 4} have been declared as UpValues for n.

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for n.

** DefTensor: Defining tensor
 ah[LI[xAct`xPand`Private`p$2742], LI[xAct`xPand`Private`q$2742]].

** DefTensor: Defining tensor
 Hh[LI[xAct`xPand`Private`p$2747], LI[xAct`xPand`Private`q$2747]].

** DefTensor: Defining symmetric metric tensor gah2[-σ$2753, -σ$2754].

** DefTensor: Defining inverse metric tensor
 Invgah2[σ$2753, σ$2754]. Metric is frozen!

** DefMetric: Don't know yet how to define epsilon for a frozen metric.

** DefCovD: Defining covariant derivative CDah2[-σ$2753].

** DefTensor: Defining vanishing torsion tensor TorsionCDah2[α, -β, -γ].

** DefTensor: Defining
 symmetric Christoffel tensor ChristoffelCDah2[α, -β, -γ].

** DefTensor: Defining Riemann tensor RiemannDownCDah2[-α, -β, -γ, -λ].

** DefTensor: Defining Riemann tensor
 RiemannCDah2[-α, -β, -γ, λ]. Antisymmetric only in the first pair.

** DefTensor: Defining symmetric Ricci tensor RicciCDah2[-α, -β].

** DefCovD:  Contractions of Riemann automatically replaced by Ricci.

** DefTensor: Defining Ricci scalar RicciScalarCDah2[].

** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.

** DefTensor: Defining symmetric Einstein tensor EinsteinCDah2[-α, -β].

** MakeRule: Potential problems moving indices on the LHS.

** DefTensor: Defining Weyl tensor WeylCDah2[-α, -β, -γ, -λ].

** DefTensor: Defining symmetric TFRicci tensor TFRicciCDah2[-α, -β].

** DefTensor: Defining Kretschmann scalar KretschmannCDah2[].

** DefCovD:  Computing RiemannToWeylRules for dim 4

** DefCovD:  Computing RicciToEinsteinRules for dim 4

** DefTensor: Defining weight +2 density Detgah2[]. Determinant.

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for g.
```

```
   Rules {1} have been declared as UpValues for g.

** DefTensor: Defining tensor Connectionh[-σ$1474, -σ$1475, -σ$1476].

** DefTensor: Defining tensor CSh[-σ$1474, -σ$1475, -σ$1476].

** DefTensor: Defining tensor nth[-σ$1474, -σ$1475].

** DefTensor: Defining tensor avh[-σ$1474].

   Rules {1, 1, 2, 1, 3, 1, 4, 1} have been declared as UpValues for av.

** DefTensor: Defining tensor Kh[LI[xAct`xPand`Private`p$3442],
  LI[xAct`xPand`Private`q$3442], -σ$1474, -σ$1475].
```

We define the metric and velocity perturbations (plus many fields which are going to be used in the splitting of these perturbations). Metric perturbations are split in line with the standard SVT decomposition.

```
In[11]:= ? DefMetricFields
         ? DefMatterFields
```

DefMetricFields[g,gpert,h,parameter] defines the perturbed metric 'gpert' from its background value 'g'. The argument 'h' is the induced metric on the spatial hypersurfaces, and the optional argument 'parameter' (set by default to ϵ) is the symbol used in the perturbative expansions. Several tensors that are used to decompose 'gpert' are defined when calling this function.

DefMatterFields[uf,ufpert,h,parameter] defines the perturbed vector field 'ufpert' from its background value 'uf'. The argument 'h' is the induced metric on the spatial hypersurfaces, and the optional argument 'parameter' (set by default to ϵ) is the symbol used in the perturbative expansions. Several tensors that are used to decompose 'ufpert' are defined when calling this function.

```
In[13]:= DefMetricFields[g, dg, h]
         DefMatterFields[u, du, h]
```

```
** DefParameter: Defining parameter ∈.

** DefTensor: Defining tensor dg[LI[order], -α, -β].

** DefTensor: Defining tensor
 ϕh[LI[xAct`xPand`Private`p$4492], LI[xAct`xPand`Private`q$4492]].

** DefTensor: Defining tensor
 Bsh[LI[xAct`xPand`Private`p$4497], LI[xAct`xPand`Private`q$4497]].

** DefTensor: Defining tensor
 Bvh[LI[xAct`xPand`Private`p$4502], LI[xAct`xPand`Private`q$4502], -α].

** DefTensor: Defining tensor
 ψh[LI[xAct`xPand`Private`p$4508], LI[xAct`xPand`Private`q$4508]].

** DefTensor: Defining tensor
 Esh[LI[xAct`xPand`Private`p$4513], LI[xAct`xPand`Private`q$4513]].

** DefTensor: Defining tensor
 Evh[LI[xAct`xPand`Private`p$4518], LI[xAct`xPand`Private`q$4518], -α].

** DefTensor: Defining tensor
 Eth[LI[xAct`xPand`Private`p$4524], LI[xAct`xPand`Private`q$4524], -α, -β].

** DefTensor: Defining tensor
 Th[LI[xAct`xPand`Private`p$4533], LI[xAct`xPand`Private`q$4533]].

** DefTensor: Defining tensor
 Lsh[LI[xAct`xPand`Private`p$4538], LI[xAct`xPand`Private`q$4538]].

** DefTensor: Defining tensor
 Lvh[LI[xAct`xPand`Private`p$4543], LI[xAct`xPand`Private`q$4543], -α].

** DefTensor: Defining tensor
 φ[LI[xAct`xPand`Private`p$4551], LI[xAct`xPand`Private`q$4551]].

** DefTensor: Defining tensor
 ρu[LI[xAct`xPand`Private`p$4556], LI[xAct`xPand`Private`q$4556]].

** DefTensor: Defining tensor
 Pu[LI[xAct`xPand`Private`p$4561], LI[xAct`xPand`Private`q$4561]].

** DefTensor: Defining tensor
 Vspathu[LI[xAct`xPand`Private`p$4566], LI[xAct`xPand`Private`q$4566], -α].

** DefTensor: Defining tensor
 V0hu[LI[xAct`xPand`Private`p$4572], LI[xAct`xPand`Private`q$4572]].

** DefTensor: Defining tensor
 Vshu[LI[xAct`xPand`Private`p$4577], LI[xAct`xPand`Private`q$4577]].

** DefTensor: Defining tensor
 Vvhu[LI[xAct`xPand`Private`p$4582], LI[xAct`xPand`Private`q$4582], -α].

** DefTensor: Defining tensor u[α].

** DefTensor: Defining tensor du[LI[order], α].
```

■ Projected tensors

DefMetricFields has defined many tensors:

In[15]:= `$Tensors`

Out[15]= {g, epsilong, Tetrag, Tetrag†, TorsionCD, ChristoffelCD, RiemannCD, RicciCD, RicciScalarCD, EinsteinCD, WeylCD, TFRicciCD, KretschmannCD, Detg, n, h, epsilonh, Tetrah, Tetrah†, Torsioncd, Christoffelcd, Riemanncd, Riccicd, RicciScalarcd, Einsteincd, Weylcd, TFRiccicd, Kretschmanncd, Deth, ExtrinsicKh, Accelerationn, ah, Hh, gah2, Invgah2, TorsionCDah2, ChristoffelCDah2, RiemannDownCDah2, RiemannCDah2, RicciCDah2, RicciScalarCDah2, EinsteinCDah2, WeylCDah2, TFRicciCDah2, KretschmannCDah2, Detgah2, Connectionh, CSh, nth, avh, Kh, dg, ϕh, Bsh, Bvh, ψh, Esh, Evh, Eth, Th, Lsh, Lvh, φ, ρu, Pu, Vspathu, V0hu, Vshu, Vvhu, u, du}

We can see that the first label index (LI[]) in these tensors is the order of perturbations, and the second one is the number of "time derivative"

In[16]:=
```
Eth[LI[1], LI[1], -μ, -ν]
LieD[n[α]]@%
PerturbationOrder[%%]
```

Out[16]= $^{(1)}E'_{\mu\nu}$

Out[17]= $^{(1)}E''_{\mu\nu}$

Out[18]= 1

For any tensor, the prime notation, means: $T_{\mu\nu}' = \mathcal{L}_n T_{\mu\nu}$, which implies for the contravariant indices: $T^{\mu\nu}{}' = g^{\alpha\mu} g^{\beta\nu} \mathcal{L}_n T_{\alpha\beta}$.

For Minkowski, FLFlat and FLCurved space-times, since $\mathcal{L}_n g_{\mu\nu} = 0$ and $\mathcal{L}_n g^{\mu\nu} = 0$, the number of "time derivative" for a contravariant tensor is also represented by prime symbols.

But for the different Bianchi cases (namely, Bianchi I, Bianchi A and Bianchi B), since $\mathcal{L}_n g_{\mu\nu} = -2\,\sigma_{\mu\nu}$ and $\mathcal{L}_n g^{\mu\nu} = -2\,\sigma^{\mu\nu}$, the number of "time derivative" for a contravariant tensor is *not* represented by prime symbols (see further below).

■ Conformal transformation

When evaluating SetSlicing, a conformal metric is defined by calling the function DefConformalMetric[g,ah]. We can check indeed it is in the list of metrics that is there is indeed a new defined metric, gah2.

In[19]:= `$Metrics`

Out[19]= {g, h, gah2}

The latter relates to the first metric g according to

```
In[20]:= ConformalRules[First@$Metrics, gah2]
         Conformal[g, gah2][g[μ, ν]]
         Conformal[g, gah2][g[-μ, -ν]]
```

Out[20]= $\left\{ \overline{\underline{\underline{g}}}^{\alpha\beta} \to \frac{[\overline{g}\, a^2]^{\alpha\beta}}{a^2},\ \overline{\underline{\underline{g}}}^{\alpha\beta} \to a^2\ \mathrm{Inv}[\overline{g}\, a^2]^{\alpha\beta} \right\}$

** DefTensor: Defining tensor ChristoffelCDCDah2[α, -β, -γ].

Out[21]= $\frac{g^{\mu\nu}}{(a)^2}$

Out[22]= $\overline{g}_{\mu\nu}\ (a)^2$

```
In[23]:= ?Conformal
```

Conformal[metricfinal][metric1,metric2][expr] performs a conformal transformation of 'expr' from 'metric1' to 'metric2' and expresses the result in terms of 'metricfinal' (along with its associated covariant derivative and curvature tensors). If needed, the function first reformulates 'expr' in terms of 'metric1'. The two metrics 'metric1' and 'metric2' have to be conformally related by DefConformalMetric.

Conformal[metric1,metric2][expr] provides the final result with respect to the ambient metric, which is the first metric defined on the manifold.

We then have a function called Conformal, which performs conformal transformations and expresses the result by default in terms of the original metric

```
In[24]:= DefTensor[Te[-α], {M}, PrintAs → "T"]
         DefTensor[Te2[-α, -β], {M}, PrintAs → "T"]
```

** DefTensor: Defining tensor Te[-α].

** DefTensor: Defining tensor Te2[-α, -β].

```
In[26]:= Conformal[g, gah2][CD[-ν][Te[-μ]]]
         Conformal[g, gah2][CD[-ν][Te2[μ, λ]]]
         Conformal[g, gah2][ChristoffelCD[α, -ν, -β]]
```

Out[26]= $\overline{g}_{\mu\nu}\ T^{\alpha}\ (\overline{\nabla}_{\alpha} a) - T_{\nu}\ (\overline{\nabla}_{\mu} a) + a\ (\overline{\nabla}_{\nu} T_{\mu})$

Out[27]= $\frac{\delta^{\mu}{}_{\nu}\ T^{\alpha\lambda}\ (\overline{\nabla}_{\alpha} a)}{(a)^3} + \frac{\delta^{\lambda}{}_{\nu}\ T^{\mu\alpha}\ (\overline{\nabla}_{\alpha} a)}{(a)^3} - \frac{T^{\mu}{}_{\nu}\ (\overline{\nabla}^{\lambda} a)}{(a)^3} - \frac{T_{\nu}{}^{\lambda}\ (\overline{\nabla}^{\mu} a)}{(a)^3} + \frac{\overline{\nabla}_{\nu} T^{\mu\lambda}}{(a)^2}$

Out[28]= $\Gamma[\overline{\nabla}]^{\alpha}{}_{\beta\nu} - \frac{\overline{g}_{\beta\nu}\ (\overline{\nabla}^{\alpha} a)}{(a)} + \frac{\delta^{\alpha}{}_{\nu}\ (\overline{\nabla}_{\beta} a)}{(a)} + \frac{\delta^{\alpha}{}_{\beta}\ (\overline{\nabla}_{\nu} a)}{(a)}$

We can also verify the conformal weight of any quantity by executing

In[29]:=
```
ConformalWeight[Te[α]]
ConformalWeight[Te[-α]]
```

Out[29]= -1

Out[30]= 1

In[31]:=
```
ConformalWeight[Te2[α, β]]
ConformalWeight[Te2[-α, -β]]
```

Out[31]= -2

Out[32]= 2

We can also perform the conformal transformation of the curvature tensors that are defined once a metric is chosen (for example, the Ricci scalar). The perturbative expansion and the conformal transformation commute.

The perturbation are performed with the xPert tools Perturbed and ExpandPerturbation

In[33]:=
```
?Perturbed
?ExpandPerturbation
```

Perturbed[expr, n] returns the expansion of expr perturbed up to n–th order, using $PerturbationParameter.

ExpandPerturbation[expr] returns expr with all Perturbation expressions expanded in terms of metric perturbations. It uses fast pre–stored formulas for the general term of the perturbative expansion of the most common curvature tensors.

```
In[35]:= Conformal[g, gah2][RicciScalarCD[]]
         AA = ExpandPerturbation@Perturbed[Conformal[g, gah2][RicciCD[-μ, -ν]], 1] // org
         BB = Conformal[g, gah2][ExpandPerturbation@Perturbed[RicciCD[-μ, -ν], 1]] // org
         org[AA - BB]
```

Out[35]=
$$\frac{R[\nabla]}{(a)^2} - \frac{6\left(\nabla_\alpha \nabla^\alpha a\right)}{(a)^3}$$

Out[36]=
$$\begin{aligned}
&R[\nabla]_{\mu\nu} - \frac{\bar{g}_{\mu\nu}\left(\nabla_\alpha\nabla^\alpha a\right)}{a} - \frac{\bar{g}_{\mu\nu}\left(\nabla_\alpha a\right)\left(\nabla^\alpha a\right)}{a^2} + \frac{4\left(\nabla_\mu a\right)\left(\nabla_\nu a\right)}{a^2} - \\
&\frac{2\left(\nabla_\nu\nabla_\mu a\right)}{a} + \epsilon\left(-\frac{\delta g^{1}{}_{\mu\nu}\left(\nabla_\alpha\nabla^\alpha a\right)}{a} - \frac{1}{2}\left(\nabla_\alpha\nabla^\alpha\delta g^{1}{}_{\mu\nu}\right) + \frac{1}{2}\left(\nabla_\alpha\nabla_\mu\delta g^{1}{}_{\nu}{}^{\alpha}\right) + \right. \\
&\frac{1}{2}\left(\nabla_\alpha\nabla_\nu\delta g^{1}{}_{\mu}{}^{\alpha}\right) - \frac{\delta g^{1}{}_{\mu\nu}\left(\nabla_\alpha a\right)\left(\nabla^\alpha a\right)}{a^2} - \frac{\bar{g}_{\mu\nu}\left(\nabla_\alpha\delta g^{1\beta}{}_{\beta}\right)\left(\nabla^\alpha a\right)}{2\,a} - \\
&\frac{\left(\nabla_\alpha\delta g^{1}{}_{\mu\nu}\right)\left(\nabla^\alpha a\right)}{a} + \frac{\delta g^{1\alpha\beta}\,\bar{g}_{\mu\nu}\left(\nabla_\alpha a\right)\left(\nabla_\beta a\right)}{a^2} + \frac{\bar{g}_{\mu\nu}\left(\nabla^\alpha a\right)\left(\nabla_\beta\delta g^{1}{}_{\alpha}{}^{\beta}\right)}{a} + \\
&\left.\frac{\delta g^{1\alpha\beta}\,\bar{g}_{\mu\nu}\left(\nabla_\beta\nabla_\alpha a\right)}{a} + \frac{\left(\nabla^\alpha a\right)\left(\nabla_\mu\delta g^{1}{}_{\nu\alpha}\right)}{a} + \frac{\left(\nabla^\alpha a\right)\left(\nabla_\nu\delta g^{1}{}_{\mu\alpha}\right)}{a} - \frac{1}{2}\left(\nabla_\nu\nabla_\mu\delta g^{1\alpha}{}_{\alpha}\right)\right)
\end{aligned}$$

Out[37]=
$$\begin{aligned}
&R[\nabla]_{\mu\nu} - \frac{\bar{g}_{\mu\nu}\left(\nabla_\alpha\nabla^\alpha a\right)}{a} - \frac{\bar{g}_{\mu\nu}\left(\nabla_\alpha a\right)\left(\nabla^\alpha a\right)}{a^2} + \frac{4\left(\nabla_\mu a\right)\left(\nabla_\nu a\right)}{a^2} - \\
&\frac{2\left(\nabla_\nu\nabla_\mu a\right)}{a} + \epsilon\left(-\frac{\delta g^{1}{}_{\mu\nu}\left(\nabla_\alpha\nabla^\alpha a\right)}{a} - \frac{1}{2}\left(\nabla_\alpha\nabla^\alpha\delta g^{1}{}_{\mu\nu}\right) + \frac{1}{2}\left(\nabla_\alpha\nabla_\mu\delta g^{1}{}_{\nu}{}^{\alpha}\right) + \right. \\
&\frac{1}{2}\left(\nabla_\alpha\nabla_\nu\delta g^{1}{}_{\mu}{}^{\alpha}\right) - \frac{\delta g^{1}{}_{\mu\nu}\left(\nabla_\alpha a\right)\left(\nabla^\alpha a\right)}{a^2} - \frac{\bar{g}_{\mu\nu}\left(\nabla_\alpha\delta g^{1\beta}{}_{\beta}\right)\left(\nabla^\alpha a\right)}{2\,a} - \\
&\frac{\left(\nabla_\alpha\delta g^{1}{}_{\mu\nu}\right)\left(\nabla^\alpha a\right)}{a} + \frac{\delta g^{1\alpha\beta}\,\bar{g}_{\mu\nu}\left(\nabla_\alpha a\right)\left(\nabla_\beta a\right)}{a^2} + \frac{\bar{g}_{\mu\nu}\left(\nabla^\alpha a\right)\left(\nabla_\beta\delta g^{1}{}_{\alpha}{}^{\beta}\right)}{a} + \\
&\left.\frac{\delta g^{1\alpha\beta}\,\bar{g}_{\mu\nu}\left(\nabla_\beta\nabla_\alpha a\right)}{a} + \frac{\left(\nabla^\alpha a\right)\left(\nabla_\mu\delta g^{1}{}_{\nu\alpha}\right)}{a} + \frac{\left(\nabla^\alpha a\right)\left(\nabla_\nu\delta g^{1}{}_{\mu\alpha}\right)}{a} - \frac{1}{2}\left(\nabla_\nu\nabla_\mu\delta g^{1\alpha}{}_{\alpha}\right)\right)
\end{aligned}$$

Out[38]= 0

General splitting of perturbations

■ Curvature fields

We first review all the steps which are necessary in order to compute cosmological perturbations.

We will then see that some functions are predefined to perform all these steps at once.

The perturbations in general are obtained by first performing a conformal transformation, and then perturbing the result with the tools of xPert (Perturbed[...,order]).

For instance the Ricci scalar can be conformally transformed, and then perturbed. We decompose these steps to see clearly what happens.

```
In[39]:= order = 1;
         RicciScalarCD[]
         Conformal[g, gah2][%]
         MyR = ExpandPerturbation@Perturbed[%, order] // org
```

Out[40]= $R[\overline{\nabla}]$

Out[41]= $$\frac{R[\overline{\nabla}]}{(a)^2} - \frac{6\,(\overline{\nabla}_\alpha \overline{\nabla}^\alpha a)}{(a)^3}$$

Out[42]= $$\frac{R[\overline{\nabla}]}{a^2} - \frac{6\,(\overline{\nabla}_\alpha \overline{\nabla}^\alpha a)}{a^3} + \epsilon \left(-\frac{\delta g^{1\alpha\beta}\; R[\overline{\nabla}]_{\alpha\beta}}{a^2} - \frac{3\,(\overline{\nabla}_\alpha \delta g^{1\beta}{}_\beta)\,(\overline{\nabla}^\alpha a)}{a^3} + \frac{6\,(\overline{\nabla}^\alpha a)\,(\overline{\nabla}_\beta \delta g^{1}{}_\alpha{}^\beta)}{a^3} + \frac{6\,\delta g^{1\alpha\beta}\,(\overline{\nabla}_\beta \overline{\nabla}_\alpha a)}{a^3} + \frac{\overline{\nabla}_\beta \overline{\nabla}_\alpha \delta g^{1\alpha\beta}}{a^2} - \frac{\overline{\nabla}_\beta \overline{\nabla}^\beta \delta g^{1\alpha}{}_\alpha}{a^2} \right)$$

We need some rules to replace the metric perturbations, and to parameterize its background 3+1 splitting. SplitMetric builds these rules for a given splitting and a given gauge. We can see all the metric variables associated with a given gauge using VisualizeTensor.

```
In[43]:= ? $ListOfGauges
```

Possible gauges are: AnyGauge, ComovingGauge, FlatGauge, IsoDensityGauge, NewtonGauge and SynchronousGauge.

```
In[44]:= ? VisualizeTensor
```

VisualizeTensor[expr,h] displays in a table the projections of 'expr' along its time components (i.e. along the vector normal to the hypersurfaces) and along its space components (i.e. onto the hypersurfaces), being defined by the background slicing associated with the induced metric 'h'.

'expr' has to be an expression involving tensors of rank 2 only.

In[45]:=
```
VisualizeTensor[dg[LI[1], μ, ν] /. SplitMetric[g, dg, h, "AnyGauge"], h]
VisualizeTensor[dg[LI[1], μ, ν] /. SplitMetric[g, dg, h, "ComovingGauge"], h]
VisualizeTensor[dg[LI[1], μ, ν] /. SplitMetric[g, dg, h, "NewtonGauge"], h]
VisualizeTensor[dg[LI[1], μ, ν] /. SplitMetric[g, dg, h, "SynchronousGauge"], h]
```

Out[45]=

	n	h
n	$-2\left(\overset{(1)}{\phi}\right)$	$-\left(\overset{(1)}{B}{}^{\nu}\right) - \overline{D}^{\nu}\,\overset{(1)}{B}$
h	$-\left(\overset{(1)}{B}{}^{\mu}\right) - \overline{D}^{\mu}\,\overset{(1)}{B}$	$2\left(\overset{(1)}{E}{}^{\mu\nu}\right) - 2\,\bar{h}^{\mu\nu}\left(\overset{(1)}{\psi}\right) + \overline{D}^{\mu}\,\overset{(1)}{E}{}^{\nu} + \overline{D}^{\nu}\,\overset{(1)}{E}{}^{\mu} + 2\left(\overline{D}^{\nu}\,\overline{D}^{\mu}\,\overset{(1)}{E}\right)$

Out[46]=

	n	h
n	$-2\left(\overset{(1)}{\phi}\right)$	$-\left(\overset{(1)}{B}{}^{\nu}\right) - \overline{D}^{\nu}\,\overset{(1)}{B}$
h	$-\left(\overset{(1)}{B}{}^{\mu}\right) - \overline{D}^{\mu}\,\overset{(1)}{B}$	$2\left(\overset{(1)}{E}{}^{\mu\nu}\right) - 2\,\bar{h}^{\mu\nu}\left(\overset{(1)}{\psi}\right)$

Out[47]=

	n	h
n	$-2\left(\overset{(1)}{\phi}\right)$	$-\left(\overset{(1)}{B}{}^{\nu}\right)$
h	$-\left(\overset{(1)}{B}{}^{\mu}\right)$	$2\left(\overset{(1)}{E}{}^{\mu\nu}\right) - 2\,\bar{h}^{\mu\nu}\left(\overset{(1)}{\psi}\right)$

Out[48]=

	n	h
n	0	0
h	0	$2\left(\overset{(1)}{E}{}^{\mu\nu}\right) - 2\,\bar{h}^{\mu\nu}\left(\overset{(1)}{\psi}\right) + \overline{D}^{\mu}\,\overset{(1)}{E}{}^{\nu} + \overline{D}^{\nu}\,\overset{(1)}{E}{}^{\mu} + 2\left(\overline{D}^{\nu}\,\overline{D}^{\mu}\,\overset{(1)}{E}\right)$

Below are Boolean operators that the user could use to set vectors and tensors to zero at first order.

In[49]:=
```
$FirstOrderVectorPerturbations = True;
$FirstOrderTensorPerturbations = True;
```

If the user does not like how the default notations are printed, it is easy to switch to one's choice. For example, the variable ϕh[LI[1], LI[2]] is printed as

In[51]:=
```
ϕh[LI[1], LI[2]]
```

Out[51]= $\overset{(1)\prime\prime}{\phi}$

Assuming we prefer \Phi, we only need to do this

In[52]:=
```
PrintAs[ϕh] ^= "Φ"
```

Out[52]= Φ

Immediately we get what we wanted

In[53]:=
```
ϕh[LI[1], LI[2]]
```

Out[53]= $\overset{(1)\prime\prime}{\Phi}$

Then we use the function SplitPerturbations which takes as arguments a perturbed expression, a list of delayed rules, and the induced metric in order to specify the background splitting.

The delayed rules are usually built through the function SplitMetric for the standard gauge choices.

In[54]:= **?SplitPerturbations**

SplitPerturbations[expr,ListPairs,h] splits the expression 'expr', using the list of rules 'ListPairs', according to the background slicing associated with the induced metric 'h'.

```
In[55]:= SplitPerturbations[MyR, SplitMetric[g, dg, h, "AnyGauge"], h]
```

The Splitting of $\frac{R[\bar{\nabla}]}{a^2}$ +... was performed in 0.853922 seconds.

Out[55]=
$$\frac{6\mathcal{H}^2}{a^2}+\frac{6\mathcal{H}'}{a^2}+\epsilon\left(-\frac{12\mathcal{H}^2\left({}^{(1)}\Phi\right)}{a^2}-\frac{12\mathcal{H}'\left({}^{(1)}\Phi\right)}{a^2}-\frac{6\mathcal{H}\left({}^{(1)}\Phi'\right)}{a^2}-\frac{18\mathcal{H}\left({}^{(1)}\psi'\right)}{a^2}-\frac{6\left({}^{(1)}\psi''\right)}{a^2}-\frac{6\mathcal{H}\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}B\right)}{a^2}-\frac{2\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}B'\right)}{a^2}+\frac{6\mathcal{H}\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}E'\right)}{a^2}+\frac{2\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}E''\right)}{a^2}-\frac{2\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}\Phi\right)}{a^2}+\frac{4\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}\psi\right)}{a^2}\right)$$

We can perform all these steps in one piece. The main four steps are:

1) Conformal transformation with Conformal,

2) Perturbation with Expandperturbation@Perturbed,

3) The creation of a rule for the metric perturbation and the matter perturbations for a given gauge choice. This is done through SplitMetric and SplitMatter

4) The splitting of the result by SplitPerturbations

The function ToxPand is designed to perform these steps at once (that's the function for users who do not want to build by themselves the rules to split the perturbations.)

It does everything for our choice of gauge and our splitting (referred to by its induced metric h), given the perturbation of the metric dg, the velocity field u (and its peturbation du):

In[56]:= ? ToxPand

xPand[expr,gpert,uf,ufpert,h,gauge,order].

This function first performs on 'expr' a conformal transformation with the scale factor 'a[h]' (except for a Minkowski 'SpaceType'); then it perturbs the result up to the order 'order' by means of xPert tools; and finally, it splits the result using the rules associated with the 'gauge', and according to the background slicing associated with the induced metric 'h'.

All the perturbed fields are derived from the perturbed metric 'gpert' and the perturbed fluid four–velocity 'ufpert'.

```
In[57]:= MyToxPand[expr_, gauge_, order_] := ToxPand[expr, dg, u, du, h, gauge, order]
```

The Ricci Scalar R

```
In[58]:= MyToxPand[RicciScalarCD[], "AnyGauge", order]
```

The Splitting of $\frac{R[\bar{\nabla}]}{(a)^2}$ +... was performed in 0.860410 seconds.

Out[58]=
$$\frac{6\,\mathcal{H}^2}{a^2}+\frac{6\,\mathcal{H}'}{a^2}+\epsilon\left(-\frac{12\,\mathcal{H}^2\left({}^{(1)}\Phi\right)}{a^2}-\frac{12\,\mathcal{H}'\left({}^{(1)}\Phi\right)}{a^2}-\frac{6\,\mathcal{H}\left({}^{(1)}\Phi'\right)}{a^2}-\frac{18\,\mathcal{H}\left({}^{(1)}\psi'\right)}{a^2}-\frac{6\left({}^{(1)}\psi''\right)}{a^2}-\frac{6\,\mathcal{H}\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}B\right)}{a^2}-\frac{2\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}B'\right)}{a^2}+\frac{6\,\mathcal{H}\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}E'\right)}{a^2}+\frac{2\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}E''\right)}{a^2}-\frac{2\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}\Phi\right)}{a^2}+\frac{4\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}\psi\right)}{a^2}\right)$$

Or we can restrict to any of the gauge choices listed below:

```
In[59]:= ? $ListOfGauges
```

Possible gauges are: AnyGauge, ComovingGauge, FlatGauge, IsoDensityGauge, NewtonGauge and SynchronousGauge.

```
In[60]:= MyToxPand[RicciScalarCD[], "NewtonGauge", order]
```

The Splitting of $\frac{R[\nabla]}{(a)^2}$ +... was performed in 0.616203 seconds.

Out[60]= $$\frac{6\,\mathcal{H}^2}{a^2}+\frac{6\,\mathcal{H}'}{a^2}+\epsilon\left(-\frac{12\,\mathcal{H}^2\,\left({}^{(1)}\Phi\right)}{a^2}-\frac{12\,\mathcal{H}'\,\left({}^{(1)}\Phi\right)}{a^2}-\frac{6\,\mathcal{H}\,\left({}^{(1)}\Phi'\right)}{a^2}-\frac{18\,\mathcal{H}\,\left({}^{(1)}\psi'\right)}{a^2}-\frac{6\,\left({}^{(1)}\psi''\right)}{a^2}-\frac{2\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}\Phi\right)}{a^2}+\frac{4\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}\psi\right)}{a^2}\right)$$

```
In[61]:= MyToxPand[RicciScalarCD[], "FlatGauge", order]
```

The Splitting of $\frac{R[\nabla]}{(a)^2}$ +... was performed in 0.552516 seconds.

Out[61]= $$\frac{6\,\mathcal{H}^2}{a^2}+\frac{6\,\mathcal{H}'}{a^2}+\epsilon\left(-\frac{12\,\mathcal{H}^2\,\left({}^{(1)}\Phi\right)}{a^2}-\frac{12\,\mathcal{H}'\,\left({}^{(1)}\Phi\right)}{a^2}-\frac{6\,\mathcal{H}\,\left({}^{(1)}\Phi'\right)}{a^2}-\frac{6\,\mathcal{H}\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}B\right)}{a^2}-\frac{2\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}B'\right)}{a^2}-\frac{2\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}\Phi\right)}{a^2}\right)$$

```
In[62]:= MyToxPand[RicciScalarCD[], "SynchronousGauge", order]
```

The Splitting of $\frac{R[\nabla]}{(a)^2}$ +... was performed in 0.745126 seconds.

Out[62]= $$\frac{6\,\mathcal{H}^2}{a^2}+\frac{6\,\mathcal{H}'}{a^2}+\epsilon\left(-\frac{18\,\mathcal{H}\,\left({}^{(1)}\psi'\right)}{a^2}-\frac{6\,\left({}^{(1)}\psi''\right)}{a^2}+\frac{6\,\mathcal{H}\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}E'\right)}{a^2}+\frac{2\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}E''\right)}{a^2}+\frac{4\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}\psi\right)}{a^2}\right)$$

```
In[63]:= MyToxPand[RicciScalarCD[], "ComovingGauge", order]
```

The Splitting of $\frac{R[\nabla]}{(a)^2}$ +... was performed in 0.637528 seconds.

Out[63]= $$\frac{6\,\mathcal{H}^2}{a^2}+\frac{6\,\mathcal{H}'}{a^2}+\epsilon\left(-\frac{12\,\mathcal{H}^2\,\left({}^{(1)}\Phi\right)}{a^2}-\frac{12\,\mathcal{H}'\,\left({}^{(1)}\Phi\right)}{a^2}-\frac{6\,\mathcal{H}\,\left({}^{(1)}\Phi'\right)}{a^2}-\frac{18\,\mathcal{H}\,\left({}^{(1)}\psi'\right)}{a^2}-\frac{6\,\left({}^{(1)}\psi''\right)}{a^2}-\frac{6\,\mathcal{H}\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}B\right)}{a^2}-\frac{2\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}B'\right)}{a^2}-\frac{2\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}\Phi\right)}{a^2}+\frac{4\,\left(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}\psi\right)}{a^2}\right)$$

The Einstein tensor $G_{\mu\nu}$ can also be obtained
(we here restrict ourrselves to the Newton gauge to avoid too large expressions).

```
In[64]:= MyToxPand[EinsteinCD[-μ, -ν], "NewtonGauge", 1]
```

The Splitting of $R[\bar{\nabla}]_{\mu\nu}$ +... was performed in 2.796074 seconds.

Out[64]=
$$-\bar{h}_{\mu\nu}\mathcal{H}^2 - 2\bar{h}_{\mu\nu}\mathcal{H}' + 3\mathcal{H}^2\bar{n}_\mu\bar{n}_\nu + \epsilon\Big({}^{(1)}E''_{\mu\nu} + 2\left({}^{(1)}E'_{\mu\nu}\right)\mathcal{H} - 2\left({}^{(1)}E_{\mu\nu}\right)\mathcal{H}^2 - 4\left({}^{(1)}E_{\mu\nu}\right)\mathcal{H}' + \left({}^{(1)}B_\nu\right)\mathcal{H}^2\bar{n}_\mu +$$
$$2\left({}^{(1)}B_\nu\right)\mathcal{H}'\bar{n}_\mu + \left({}^{(1)}B_\mu\right)\mathcal{H}^2\bar{n}_\nu + 2\left({}^{(1)}B_\mu\right)\mathcal{H}'\bar{n}_\nu + 2\bar{h}_{\mu\nu}\mathcal{H}^2\left({}^{(1)}\Phi\right) +$$
$$4\bar{h}_{\mu\nu}\mathcal{H}'\left({}^{(1)}\Phi\right) + 2\bar{h}_{\mu\nu}\mathcal{H}\left({}^{(1)}\Phi'\right) + 2\bar{h}_{\mu\nu}\mathcal{H}^2\left({}^{(1)}\psi\right) + 4\bar{h}_{\mu\nu}\mathcal{H}'\left({}^{(1)}\psi\right) + 4\bar{h}_{\mu\nu}\mathcal{H}\left({}^{(1)}\psi'\right) -$$
$$6\mathcal{H}\bar{n}_\mu\bar{n}_\nu\left({}^{(1)}\psi'\right) + 2\bar{h}_{\mu\nu}\left({}^{(1)}\psi''\right) + \frac{1}{2}\bar{n}_\nu\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}B_\mu\right) + \frac{1}{2}\bar{n}_\mu\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}B_\nu\right) -$$
$$\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}E_{\mu\nu} + \bar{h}_{\mu\nu}\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}\Phi\right) - \bar{h}_{\mu\nu}\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}\psi\right) + 2\bar{n}_\mu\bar{n}_\nu\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}\psi\right) -$$
$$\mathcal{H}\left(\bar{D}_\mu\,{}^{(1)}B_\nu\right) - \frac{1}{2}\left(\bar{D}_\mu\,{}^{(1)}B'_\nu\right) - 2\mathcal{H}\bar{n}_\nu\left(\bar{D}_\mu\,{}^{(1)}\Phi\right) - 2\bar{n}_\nu\left(\bar{D}_\mu\,{}^{(1)}\psi'\right) - \mathcal{H}\left(\bar{D}_\nu\,{}^{(1)}B_\mu\right) -$$
$$\frac{1}{2}\left(\bar{D}_\nu\,{}^{(1)}B'_\mu\right) - 2\mathcal{H}\bar{n}_\mu\left(\bar{D}_\nu\,{}^{(1)}\Phi\right) - 2\bar{n}_\mu\left(\bar{D}_\nu\,{}^{(1)}\psi'\right) - \bar{D}_\nu\bar{D}_\mu\,{}^{(1)}\Phi + \bar{D}_\nu\bar{D}_\mu\,{}^{(1)}\psi\Big)$$

From which we can extract for instance the 00 part just by

```
In[65]:= org[% n[μ] n[ν]]
```

Out[65]=
$$3\mathcal{H}^2 + \epsilon\left(-6\mathcal{H}\left({}^{(1)}\psi'\right) + 2\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}\psi\right)\right)$$

or by using the xPand built-in function

```
In[66]:= ?ExtractComponents
```

ExtractComponents[expr,h,ListOfProjectors] projects 'expr' along its time component(s) (i.e. along the vector normal to the hypersurfaces) and along its space component(s) (i.e. onto the hypersurfaces), being defined by the background slicing associated with the induced metric 'h'.

'ListOfProjectors' is a list whose elements are among: 'Time' (or, equivalently, '∗NameOfNormalVector∗') and 'Space' (or, equivalently, '∗NameOfInducedMetric∗'). The length of the list has to equal the rank of the tensors of 'expr'. The free indices (in canonical order) of 'expr' are then projected according to the 'ListOfProjectors'.

When the argument 'ListOfProjectors' is omitted, all the projections for rank−0 to rank−2 tensors are displayed in a table.

```
In[67]:= ExtractComponents[%%%, h, {"Time", "Time"}]
```

Out[67]= $3\mathcal{H}^2 + \epsilon\left(-6\,\mathcal{H}\left({}^{(1)}\psi'\right) + 2\left(\bar{D}_\alpha\,\bar{D}^\alpha\;{}^{(1)}\psi\right)\right)$

and the (ij) component

```
In[68]:= ExtractComponents[%%%%, h, {"Space", "Space"}]
```

Out[68]= $-\bar{h}_{\mu\nu}\,\mathcal{H}^2 - 2\,\bar{h}_{\mu\nu}\,\mathcal{H}' + \epsilon\Big({}^{(1)}E''_{\mu\nu} + 2\left({}^{(1)}E'_{\mu\nu}\right)\mathcal{H} - 2\left({}^{(1)}E_{\mu\nu}\right)\mathcal{H}^2 - 4\left({}^{(1)}E_{\mu\nu}\right)\mathcal{H}' + 2\,\bar{h}_{\mu\nu}\,\mathcal{H}^2\left({}^{(1)}\Phi\right) + 4\,\bar{h}_{\mu\nu}\,\mathcal{H}'\left({}^{(1)}\Phi\right) + 2\,\bar{h}_{\mu\nu}\,\mathcal{H}\left({}^{(1)}\Phi'\right) + 2\,\bar{h}_{\mu\nu}\,\mathcal{H}^2\left({}^{(1)}\psi\right) + 4\,\bar{h}_{\mu\nu}\,\mathcal{H}'\left({}^{(1)}\psi\right) + 4\,\bar{h}_{\mu\nu}\,\mathcal{H}\left({}^{(1)}\psi'\right) + 2\,\bar{h}_{\mu\nu}\left({}^{(1)}\psi''\right) - \bar{D}_\alpha\,\bar{D}^\alpha\;{}^{(1)}E_{\mu\nu} + \bar{h}_{\mu\nu}\left(\bar{D}_\alpha\,\bar{D}^\alpha\;{}^{(1)}\Phi\right) - \bar{h}_{\mu\nu}\left(\bar{D}_\alpha\,\bar{D}^\alpha\;{}^{(1)}\psi\right) - \mathcal{H}\left(\bar{D}_\mu\;{}^{(1)}B_\nu\right) - \frac{1}{2}\left(\bar{D}_\mu\;{}^{(1)}B'_\nu\right) - \mathcal{H}\left(\bar{D}_\nu\;{}^{(1)}B_\mu\right) - \frac{1}{2}\left(\bar{D}_\nu\;{}^{(1)}B'_\mu\right) - \bar{D}_\nu\,\bar{D}_\mu\;{}^{(1)}\Phi + \bar{D}_\nu\,\bar{D}_\mu\;{}^{(1)}\psi\Big)$

It is also possible to extract all the components at once and have the final result presented in grid form.

```
In[69]:= MyToxPand[EinsteinCD[-β, -α], "NewtonGauge", 1];
         ExtractComponents[%, h]
```

The Splitting of $R[\bar{\nabla}]_{\alpha\beta}$ +... was performed in 2.935532 seconds.

Out[70]=

	n	h
n	$3\mathcal{H}^2 + \epsilon\left(-6\mathcal{H}\left({}^{(1)}\psi'\right) + 2\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\psi\right)\right)$	$\epsilon\left(-\left({}^{(1)}B_\beta\right)\mathcal{H}^2 - 2\left({}^{(1)}B_\beta\right)\mathcal{H}' + 2\mathcal{H}\left(\bar{D}_\beta\,{}^{(1)}\Phi\right) + 2\left(\bar{D}_\beta\,{}^{(1)}\psi'\right) - \frac{1}{2}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}B_\beta\right)\right)$
h	$\epsilon\left(-\left({}^{(1)}B_\alpha\right)\mathcal{H}^2 - 2\left({}^{(1)}B_\alpha\right)\mathcal{H}' + 2\mathcal{H}\left(\bar{D}_\alpha\,{}^{(1)}\Phi\right) + 2\left(\bar{D}_\alpha\,{}^{(1)}\psi'\right) - \frac{1}{2}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}B_\alpha\right)\right)$	$-\bar{h}_{\alpha\beta}\mathcal{H}^2 - 2\bar{h}_{\alpha\beta}\mathcal{H}' + \epsilon\left({}^{(1)}E''_{\alpha\beta} + 2\left({}^{(1)}E'_{\alpha\beta}\right)\mathcal{H} - 2\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}^2 - 4\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}' + 2\bar{h}_{\alpha\beta}\mathcal{H}^2\left({}^{(1)}\Phi\right) + 4\bar{h}_{\alpha\beta}\mathcal{H}'\left({}^{(1)}\Phi\right) + 2\bar{h}_{\alpha\beta}\mathcal{H}\left({}^{(1)}\Phi'\right) + 2\bar{h}_{\alpha\beta}\mathcal{H}^2\left({}^{(1)}\psi\right) + 4\bar{h}_{\alpha\beta}\mathcal{H}'\left({}^{(1)}\psi\right) + 4\bar{h}_{\alpha\beta}\mathcal{H}\left({}^{(1)}\psi'\right) + 2\bar{h}_{\alpha\beta}\left({}^{(1)}\psi''\right) - \mathcal{H}\left(\bar{D}_\alpha\,{}^{(1)}B_\beta\right) - \frac{1}{2}\left(\bar{D}_\alpha\,{}^{(1)}B'_\beta\right) - \mathcal{H}\left(\bar{D}_\beta\,{}^{(1)}B_\alpha\right) - \frac{1}{2}\left(\bar{D}_\beta\,{}^{(1)}B'_\alpha\right) - \bar{D}_\beta\bar{D}_\alpha\,{}^{(1)}\Phi + \bar{D}_\beta\bar{D}_\alpha\,{}^{(1)}\psi - \bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}E_{\alpha\beta} + \bar{h}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\Phi\right) - \bar{h}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\psi\right)\right)$

In case you want to see just a specific order, say second order, then the function ExtractOrder can be helpful.

```
In[71]:= ?ExtractOrder
```

ExtractOrder[expr,order] displays the expression at order 'order' of 'expr'.

```
In[72]:= MyToxPand[EinsteinCD[-β, -α], "NewtonGauge", 1];
         ExtractComponents[ExtractOrder[%, 1], h]
```

The Splitting of $R[\bar{\nabla}]_{\alpha\beta}$ +... was performed in 2.875676 seconds.

Out[73]=

	n	h
n	$-6\mathcal{H}\left({}^{(1)}\psi'\right)+2\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\psi\right)$	$-\left({}^{(1)}B_\beta\right)\mathcal{H}^2-2\left({}^{(1)}B_\beta\right)\mathcal{H}'+2\mathcal{H}\left(\bar{D}_\beta\,{}^{(1)}\Phi\right)+2\left(\bar{D}_\beta\,{}^{(1)}\psi'\right)-\frac{1}{2}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}B_\beta\right)$
h	$-\left({}^{(1)}B_\alpha\right)\mathcal{H}^2-2\left({}^{(1)}B_\alpha\right)\mathcal{H}'+2\mathcal{H}\left(\bar{D}_\alpha\,{}^{(1)}\Phi\right)+2\left(\bar{D}_\alpha\,{}^{(1)}\psi'\right)-\frac{1}{2}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}B_\alpha\right)$	${}^{(1)}E''_{\alpha\beta}+2\left({}^{(1)}E'_{\alpha\beta}\right)\mathcal{H}-2\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}^2-4\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}'+2\bar{h}_{\alpha\beta}\mathcal{H}^2\left({}^{(1)}\Phi\right)+4\bar{h}_{\alpha\beta}\mathcal{H}'\left({}^{(1)}\Phi\right)+2\bar{h}_{\alpha\beta}\mathcal{H}\left({}^{(1)}\Phi'\right)+2\bar{h}_{\alpha\beta}\mathcal{H}^2\left({}^{(1)}\psi\right)+4\bar{h}_{\alpha\beta}\mathcal{H}'\left({}^{(1)}\psi\right)+4\bar{h}_{\alpha\beta}\mathcal{H}\left({}^{(1)}\psi'\right)+2\bar{h}_{\alpha\beta}\left({}^{(1)}\psi''\right)-\mathcal{H}\left(\bar{D}_\alpha\,{}^{(1)}B_\beta\right)-\frac{1}{2}\left(\bar{D}_\alpha\,{}^{(1)}B'_\beta\right)-\mathcal{H}\left(\bar{D}_\beta\,{}^{(1)}B_\alpha\right)-\frac{1}{2}\left(\bar{D}_\beta\,{}^{(1)}B'_\alpha\right)-\bar{D}_\beta\bar{D}_\alpha\,{}^{(1)}\Phi+\bar{D}_\beta\bar{D}_\alpha\,{}^{(1)}\psi-\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}E_{\alpha\beta}+\bar{h}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\Phi\right)-\bar{h}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\psi\right)$

The trace-free part is extracted using some xTensor's functions.

```
In[74]:= STFPart[Projectorh[MyToxPand[EinsteinCD[-β, -α], "NewtonGauge", 1]], h] // org
```

The Splitting of $R[\bar{\nabla}]_{\alpha\beta}$ +... was performed in 2.759485 seconds.

Out[74]=

$$\epsilon\left({}^{(1)}E''_{\alpha\beta}+2\left({}^{(1)}E'_{\alpha\beta}\right)\mathcal{H}-2\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}^2-4\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}'-\mathcal{H}\left(\bar{D}_\alpha\,{}^{(1)}B_\beta\right)-\frac{1}{2}\left(\bar{D}_\alpha\,{}^{(1)}B'_\beta\right)-\mathcal{H}\left(\bar{D}_\beta\,{}^{(1)}B_\alpha\right)-\frac{1}{2}\left(\bar{D}_\beta\,{}^{(1)}B'_\alpha\right)-\bar{D}_\beta\bar{D}_\alpha\,{}^{(1)}\Phi+\bar{D}_\beta\bar{D}_\alpha\,{}^{(1)}\psi-\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}E_{\alpha\beta}+\frac{1}{3}\bar{h}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\Phi\right)-\frac{1}{3}\bar{h}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\psi\right)\right)$$

```
In[75]:= STFPart[MyToxPand[EinsteinCD[-β, -α], "NewtonGauge", 1], g] // org
```

The Splitting of $R[\bar{\nabla}]_{\alpha\beta}$ +... was performed in 2.747114 seconds.

Out[75]=

$$
\begin{aligned}
&\frac{3}{2}\bar{g}_{\alpha\beta}\mathcal{H}^2-\bar{h}_{\alpha\beta}\mathcal{H}^2+\frac{3}{2}\bar{g}_{\alpha\beta}\mathcal{H}'-2\bar{h}_{\alpha\beta}\mathcal{H}'+3\mathcal{H}^2\bar{n}_\alpha\bar{n}_\beta+\\
&\epsilon\Big({}^{(1)}E''_{\alpha\beta}+2\left({}^{(1)}E'_{\alpha\beta}\right)\mathcal{H}-2\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}^2-4\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}'+\left({}^{(1)}B_\beta\right)\mathcal{H}^2\bar{n}_\alpha+\\
&2\left({}^{(1)}B_\beta\right)\mathcal{H}'\bar{n}_\alpha+\left({}^{(1)}B_\alpha\right)\mathcal{H}^2\bar{n}_\beta+2\left({}^{(1)}B_\alpha\right)\mathcal{H}'\bar{n}_\beta-\frac{3}{2}\bar{g}_{\alpha\beta}\mathcal{H}^2\left({}^{(1)}\Phi\right)+\\
&2\bar{h}_{\alpha\beta}\mathcal{H}^2\left({}^{(1)}\Phi\right)-3\bar{g}_{\alpha\beta}\mathcal{H}'\left({}^{(1)}\Phi\right)+4\bar{h}_{\alpha\beta}\mathcal{H}'\left({}^{(1)}\Phi\right)-\frac{3}{2}\bar{g}_{\alpha\beta}\mathcal{H}\left({}^{(1)}\Phi'\right)+2\bar{h}_{\alpha\beta}\mathcal{H}\left({}^{(1)}\Phi'\right)-\\
&\frac{3}{2}\bar{g}_{\alpha\beta}\mathcal{H}^2\left({}^{(1)}\psi\right)+2\bar{h}_{\alpha\beta}\mathcal{H}^2\left({}^{(1)}\psi\right)-3\bar{g}_{\alpha\beta}\mathcal{H}'\left({}^{(1)}\psi\right)+4\bar{h}_{\alpha\beta}\mathcal{H}'\left({}^{(1)}\psi\right)-\\
&\frac{9}{2}\bar{g}_{\alpha\beta}\mathcal{H}\left({}^{(1)}\psi'\right)+4\bar{h}_{\alpha\beta}\mathcal{H}\left({}^{(1)}\psi'\right)-6\mathcal{H}\bar{n}_\alpha\bar{n}_\beta\left({}^{(1)}\psi'\right)-\frac{3}{2}\bar{g}_{\alpha\beta}\left({}^{(1)}\psi''\right)+\\
&2\bar{h}_{\alpha\beta}\left({}^{(1)}\psi''\right)-\mathcal{H}\left(\bar{D}_\alpha\,{}^{(1)}B_\beta\right)-\frac{1}{2}\left(\bar{D}_\alpha\,{}^{(1)}B'_\beta\right)-2\mathcal{H}\bar{n}_\beta\left(\bar{D}_\alpha\,{}^{(1)}\Phi\right)-2\bar{n}_\beta\left(\bar{D}_\alpha\,{}^{(1)}\psi'\right)-\\
&\mathcal{H}\left(\bar{D}_\beta\,{}^{(1)}B_\alpha\right)-\frac{1}{2}\left(\bar{D}_\beta\,{}^{(1)}B'_\alpha\right)-2\mathcal{H}\bar{n}_\alpha\left(\bar{D}_\beta\,{}^{(1)}\Phi\right)-2\bar{n}_\alpha\left(\bar{D}_\beta\,{}^{(1)}\psi'\right)-\bar{D}_\beta\bar{D}_\alpha\,{}^{(1)}\Phi+\bar{D}_\beta\bar{D}_\alpha\,{}^{(1)}\psi+\\
&\frac{1}{2}\bar{n}_\beta\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}B_\alpha\right)+\frac{1}{2}\bar{n}_\alpha\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}B_\beta\right)-\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}E_{\alpha\beta}-\frac{1}{2}\bar{g}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\Phi\right)+\\
&\bar{h}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\Phi\right)+\bar{g}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\psi\right)-\bar{h}_{\alpha\beta}\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\psi\right)+2\bar{n}_\alpha\bar{n}_\beta\left(\bar{D}_\gamma\bar{D}^\gamma\,{}^{(1)}\psi\right)\Big)
\end{aligned}
$$

■ Matter fields

General set up for the fluid velocity

Similarly to SplitMetric, we have a function SplitMatter which generates the rules for velocity perturbations, and this can then be passed as an argument to SplitPerturbations.

We can gather the rules for the metric and the rules for the matter in one list:

```
In[76]:= Rulesmetricmatter[gauge_, order_] :=
          Join[SplitMetric[g, dg, h, gauge], SplitMatter[u, du, -1, h, gauge, order]]
         Rulesmetricmatter["ComovingGauge", 1]
```

Out[77]= $\{\delta g^{1\underline{\alpha\beta}} \to 2\left({}^{(1)}E^{\alpha\beta}\right) - \left({}^{(1)}B^{\beta}\right)\bar{n}^{\alpha} - \left({}^{(1)}B^{\alpha}\right)\bar{n}^{\beta} - 2\,\bar{n}^{\alpha}\,\bar{n}^{\beta}\left({}^{(1)}\Phi\right) - 2\,\bar{h}^{\alpha\beta}\left({}^{(1)}\psi\right) - \bar{n}^{\beta}\left(\bar{D}^{\alpha}\,{}^{(1)}B\right) - \bar{n}^{\alpha}\left(\bar{D}^{\beta}\,{}^{(1)}B\right),$

$\delta g^{(\text{xAct`xPand`Private`n\$\_})?(\#1 >= 2\ \&)\underline{\alpha\beta}} \to 2\,E^{\text{xAct`xPand`Private`n\$0}\alpha\beta} - B^{\text{xAct`xPand`Private`n\$0}\beta}\,\bar{n}^{\alpha} - B^{\text{xAct`xPand`Private`n\$0}\alpha}\,\bar{n}^{\beta} - 2\,\bar{n}^{\alpha}\,\bar{n}^{\beta}\,\Phi^{\text{xAct`xPand`Private`n\$0}} - 2\,\bar{h}^{\alpha\beta}\,\psi^{\text{xAct`xPand`Private`n\$0}} - \bar{n}^{\beta}\left(\bar{D}^{\alpha}\,B^{\text{xAct`xPand`Private`n\$0}}\right) - \bar{n}^{\alpha}\left(\bar{D}^{\beta}\,B^{\text{xAct`xPand`Private`n\$0}}\right),$

$\varphi^{(\text{xAct`xPand`Private`n\_})?(\#1 >= 1\ \&)\,\text{xAct`xPand`Private`q\_}} :\to 0,$

$\text{HoldPattern}\left[u^{\underline{\underline{\alpha}}}\right] :\to \text{Module}\left[\{\}, \bar{n}^{\alpha}\right],$

$\text{HoldPattern}\left[\delta u^{1\underline{\underline{\alpha}}}\right] :\to \text{Module}\left[\{\}, \frac{1}{2}\,\bar{n}^{\alpha}\left(\delta g^{1}{}_{\alpha\beta}\,\bar{n}^{\alpha}\,\bar{n}^{\beta}\right) + {}^{(1)}V^{\alpha} - \bar{D}^{\alpha}\,{}^{(1)}B\right]\}$

But again it is much simpler to use the function MyToxPand (based on ToxPand) which does everything.

```
In[78]:= MyToxPand[u[v], "NewtonGauge", order]
         ExtractComponents[%, h]
```

The Splitting of $\frac{\epsilon\ \delta u^{1\nu}}{(a)}$ +... was performed in 0.092187 seconds.

Out[78]= $\frac{\bar{n}^{\nu}}{a} + \epsilon\left(\frac{{}^{(1)}V^{\nu}}{a} - \frac{\bar{n}^{\nu}\left({}^{(1)}\Phi\right)}{a} + \frac{\bar{D}^{\nu}\,{}^{(1)}V}{a}\right)$

Out[79]=

n	$\frac{1}{a} - \frac{\epsilon\left({}^{(1)}\Phi\right)}{a}$
h	$\epsilon\left(\frac{{}^{(1)}V^{\nu}}{a} + \frac{\bar{D}^{\nu}\,{}^{(1)}V}{a}\right)$

Or we can extract only the time part using

```
In[80]:= MyToxPand[u[ν], "NewtonGauge", order]
         ExtractComponents[%, h, {"Time"}]
```

The Splitting of $\frac{\epsilon\, \delta u^{1\nu}}{(a)}$ +... was performed in 0.091508 seconds.

Out[80]= $\frac{\bar{n}^{\nu}}{a}+\epsilon\left(\frac{{}^{(1)}V^{\nu}}{a}-\frac{\bar{n}^{\nu}\,({}^{(1)}\Phi)}{a}+\frac{\bar{D}^{\nu}\,{}^{(1)}V}{a}\right)$

Out[81]= $\frac{1}{a}-\frac{\epsilon\,({}^{(1)}\Phi)}{a}$

Divergence of the velocity field : $\nabla_{\mu}\, u^{\mu}$

```
In[82]:= MyToxPand[CD[-ν]@u[ν], "NewtonGauge", order]
```

The Splitting of $\frac{3\, u^{\nu}\,(\bar{\nabla}_{\nu} a)}{(a)^{2}}$ +... was performed in 0.215216 seconds.

Out[82]= $\frac{3\,\mathcal{H}}{a}+\epsilon\left(-\frac{3\,\mathcal{H}\,({}^{(1)}\Phi)}{a}-\frac{3\,({}^{(1)}\psi')}{a}+\frac{\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}V}{a}\right)$

Acceleration : $a^{\nu} = u^{\mu}\, \nabla_{\mu}\, u^{\nu}$

```
In[83]:= MyToxPand[u[μ] CD[-μ]@u[ν], "NewtonGauge", order]
```

The Splitting of $\frac{u^{\mu}\;u^{\nu}\,(\bar{\nabla}_{\mu} a)}{(a)^{3}}$ +... was performed in 0.435683 seconds.

Out[83]= $\epsilon\left(\frac{{}^{(1)}B'^{\nu}}{a^{2}}+\frac{({}^{(1)}B^{\nu})\,\mathcal{H}}{a^{2}}+\frac{\mathcal{H}\,({}^{(1)}V^{\nu})}{a^{2}}+\frac{{}^{(1)}V'^{\nu}}{a^{2}}+\frac{\mathcal{H}\,(\bar{D}^{\nu}\,{}^{(1)}V)}{a^{2}}+\frac{\bar{D}^{\nu}\,{}^{(1)}V'}{a^{2}}+\frac{\bar{D}^{\nu}\,{}^{(1)}\Phi}{a^{2}}\right)$

```
In[84]:= order = 2
```

Out[84]= 2

The shear of the velocity field:

```
In[85]:= MyToxPand[STFPart[Projectorh[CD[-α][u[-β]]] /. Projectorh → ProjectWith[h], h],
           "NewtonGauge", order];
         DD = ExtractComponents[%, h, {"Space", "Space"}]
```

The Splitting of $-\frac{1}{2}\, u_\beta\, \left(\bar{\nabla}_\alpha a\right)$ +... was performed in 8.863565 seconds.

Out[86]=

$$
\begin{aligned}
&\epsilon \Bigg(a \left({}^{(1)}E'_{\alpha\beta}\right) - \frac{2}{3}\, a\, \bar{h}_{\alpha\beta}\, \mathcal{H} \left({}^{(1)}\Phi\right) + \\
&\quad \frac{1}{2}\, a \left(\bar{D}_\alpha\, {}^{(1)}V_\beta\right) + \frac{1}{2}\, a \left(\bar{D}_\beta\, {}^{(1)}V_\alpha\right) + a \left(\bar{D}_\beta \bar{D}_\alpha\, {}^{(1)}V\right) - \frac{1}{3}\, a\, \bar{h}_{\alpha\beta} \left(\bar{D}_\gamma \bar{D}^\gamma\, {}^{(1)}V\right) \Bigg) + \\
&\epsilon^2 \Bigg(\frac{1}{2}\, a \left({}^{(2)}E'_{\alpha\beta}\right) - \frac{1}{3}\, a \left({}^{(1)}B^\gamma\right) \left({}^{(1)}B'_\gamma\right) \bar{h}_{\alpha\beta} + \frac{2}{3}\, a \left({}^{(1)}E^{\gamma\lambda}\right) \left({}^{(1)}E'_{\gamma\lambda}\right) \bar{h}_{\alpha\beta} + \\
&\quad \frac{1}{3}\, a \left({}^{(1)}B^\gamma\right) \bar{h}_{\alpha\beta}\, \mathcal{H} \left({}^{(1)}V_\gamma\right) - \frac{1}{3}\, a \left({}^{(1)}B^\gamma\right) \bar{h}_{\alpha\beta} \left({}^{(1)}V'_\gamma\right) - a \left({}^{(1)}E'_{\alpha\beta}\right) \left({}^{(1)}\Phi\right) - \\
&\quad \frac{4}{3}\, a \left({}^{(1)}E_{\alpha\beta}\right) \mathcal{H} \left({}^{(1)}\Phi\right) + \frac{2}{3}\, a\, \bar{h}_{\alpha\beta}\, \mathcal{H} \left({}^{(1)}\Phi\right)^2 - \frac{1}{3}\, a\, \bar{h}_{\alpha\beta}\, \mathcal{H} \left({}^{(2)}\Phi\right) + \frac{4}{3}\, a\, \bar{h}_{\alpha\beta}\, \mathcal{H} \left({}^{(1)}\Phi\right) \left({}^{(1)}\psi\right) + \\
&\quad 2\, a \left({}^{(1)}E_{\alpha\beta}\right) \left({}^{(1)}\psi'\right) - a \left({}^{(1)}\psi\right) \left(\bar{D}_\alpha\, {}^{(1)}V_\beta\right) + a \left({}^{(1)}E_\beta{}^\gamma\right) \left(\bar{D}_\alpha\, {}^{(1)}V_\gamma\right) + \frac{1}{4}\, a \left(\bar{D}_\alpha\, {}^{(2)}V_\beta\right) - \\
&\quad \frac{1}{2}\, a \left({}^{(1)}B_\beta\right) \left(\bar{D}_\alpha\, {}^{(1)}\Phi\right) - a \left({}^{(1)}\psi\right) \left(\bar{D}_\beta\, {}^{(1)}V_\alpha\right) + a \left({}^{(1)}E_\alpha{}^\gamma\right) \left(\bar{D}_\beta\, {}^{(1)}V_\gamma\right) + \\
&\quad \frac{1}{4}\, a \left(\bar{D}_\beta\, {}^{(2)}V_\alpha\right) - \frac{1}{2}\, a \left({}^{(1)}B_\alpha\right) \left(\bar{D}_\beta\, {}^{(1)}\Phi\right) - 2\, a \left({}^{(1)}\psi\right) \left(\bar{D}_\beta \bar{D}_\alpha\, {}^{(1)}V\right) + \frac{1}{2}\, a \left(\bar{D}_\beta \bar{D}_\alpha\, {}^{(2)}V\right) + \\
&\quad a \left({}^{(1)}V^\gamma\right) \left(\bar{D}_\gamma\, {}^{(1)}E_{\alpha\beta}\right) + \frac{1}{3}\, a \left({}^{(1)}B^\gamma\right) \bar{h}_{\alpha\beta}\, \mathcal{H} \left(\bar{D}_\gamma\, {}^{(1)}V\right) - \frac{1}{3}\, a \left({}^{(1)}B^\gamma\right) \bar{h}_{\alpha\beta} \left(\bar{D}_\gamma\, {}^{(1)}V'\right) + \\
&\quad a \left({}^{(1)}E_\beta{}^\gamma\right) \left(\bar{D}_\gamma \bar{D}_\alpha\, {}^{(1)}V\right) + a \left({}^{(1)}E_\alpha{}^\gamma\right) \left(\bar{D}_\gamma \bar{D}_\beta\, {}^{(1)}V\right) - \frac{2}{3}\, a \left({}^{(1)}E_{\alpha\beta}\right) \left(\bar{D}_\gamma \bar{D}^\gamma\, {}^{(1)}V\right) + \\
&\quad \frac{2}{3}\, a\, \bar{h}_{\alpha\beta} \left({}^{(1)}\psi\right) \left(\bar{D}_\gamma \bar{D}^\gamma\, {}^{(1)}V\right) - \frac{1}{6}\, a\, \bar{h}_{\alpha\beta} \left(\bar{D}_\gamma \bar{D}^\gamma\, {}^{(2)}V\right) + a \left(\bar{D}_\gamma\, {}^{(1)}E_{\alpha\beta}\right) \left(\bar{D}^\gamma\, {}^{(1)}V\right) \Bigg)
\end{aligned}
$$

```
In[87]:= EE = Projectorh[MyToxPand[STFPart[CD[-α][u[-β]], h], "NewtonGauge", order]] /.
            Projectorh → ProjectWith[h] // org
```

The Splitting of $-\frac{1}{2}\, u_\beta\, (\bar\nabla_\alpha a)$ +... was performed in 9.150923 seconds.

Out[87]=
$$\epsilon \Big(a\, ({}^{(1)}E_{\alpha\beta})' - \frac{2}{3}\, a\, \bar h_{\alpha\beta}\, \mathcal{H}\, ({}^{(1)}\Phi) + \frac{1}{2}\, a\, (\bar D_\alpha {}^{(1)}V_\beta) + \frac{1}{2}\, a\, (\bar D_\beta {}^{(1)}V_\alpha) + a\, (\bar D_\beta \bar D_\alpha {}^{(1)}V) - \frac{1}{3}\, a\, \bar h_{\alpha\beta}\, (\bar D_\gamma \bar D^\gamma {}^{(1)}V) \Big) +$$
$$\epsilon^2 \Big(\frac{1}{2}\, a\, ({}^{(2)}E_{\alpha\beta})' - \frac{1}{3}\, a\, ({}^{(1)}B^\lambda)\, ({}^{(1)}B_\lambda)'\, \bar h_{\alpha\beta} + \frac{2}{3}\, a\, ({}^{(1)}E^{\mu\nu})\, ({}^{(1)}E_{\mu\nu})'\, \bar h_{\alpha\beta} +$$
$$\frac{1}{3}\, a\, ({}^{(1)}B^\sigma)\, \bar h_{\alpha\beta}\, \mathcal{H}\, ({}^{(1)}V_\sigma) - \frac{1}{3}\, a\, ({}^{(1)}B^{\sigma 1})\, \bar h_{\alpha\beta}\, ({}^{(1)}V_{\sigma 1})' - a\, ({}^{(1)}E_{\alpha\beta})'\, ({}^{(1)}\Phi) -$$
$$\frac{4}{3}\, a\, ({}^{(1)}E_{\alpha\beta})\, \mathcal{H}\, ({}^{(1)}\Phi) + \frac{2}{3}\, a\, \bar h_{\alpha\beta}\, \mathcal{H}\, ({}^{(1)}\Phi)^2 - \frac{1}{3}\, a\, \bar h_{\alpha\beta}\, \mathcal{H}\, ({}^{(2)}\Phi) + \frac{4}{3}\, a\, \bar h_{\alpha\beta}\, \mathcal{H}\, ({}^{(1)}\Phi)\, ({}^{(1)}\psi) +$$
$$2\, a\, ({}^{(1)}E_{\alpha\beta})\, ({}^{(1)}\psi)' - a\, ({}^{(1)}\psi)\, (\bar D_\alpha {}^{(1)}V_\beta) + a\, ({}^{(1)}E_\beta{}^\gamma)\, (\bar D_\alpha {}^{(1)}V_\gamma) + \frac{1}{4}\, a\, (\bar D_\alpha {}^{(2)}V_\beta) -$$
$$\frac{1}{2}\, a\, ({}^{(1)}B_\beta)\, (\bar D_\alpha {}^{(1)}\Phi) - a\, ({}^{(1)}\psi)\, (\bar D_\beta {}^{(1)}V_\alpha) + a\, ({}^{(1)}E_\alpha{}^\gamma)\, (\bar D_\beta {}^{(1)}V_\gamma) +$$
$$\frac{1}{4}\, a\, (\bar D_\beta {}^{(2)}V_\alpha) - \frac{1}{2}\, a\, ({}^{(1)}B_\alpha)\, (\bar D_\beta {}^{(1)}\Phi) - 2\, a\, ({}^{(1)}\psi)\, (\bar D_\beta \bar D_\alpha {}^{(1)}V) + \frac{1}{2}\, a\, (\bar D_\beta \bar D_\alpha {}^{(2)}V) +$$
$$a\, ({}^{(1)}V^\gamma)\, (\bar D_\gamma {}^{(1)}E_{\alpha\beta}) + a\, ({}^{(1)}E_\beta{}^\gamma)\, (\bar D_\gamma \bar D_\alpha {}^{(1)}V) + a\, ({}^{(1)}E_\alpha{}^\gamma)\, (\bar D_\gamma \bar D_\beta {}^{(1)}V) +$$
$$a\, (\bar D_\gamma {}^{(1)}E_{\alpha\beta})\, (\bar D^\gamma {}^{(1)}V) + \frac{1}{3}\, a\, ({}^{(1)}B^{\sigma 2})\, \bar h_{\alpha\beta}\, \mathcal{H}\, (\bar D_{\sigma 2} {}^{(1)}V) - \frac{1}{3}\, a\, ({}^{(1)}B^{\sigma 3})\, \bar h_{\alpha\beta}\, (\bar D_{\sigma 3} {}^{(1)}V)' -$$
$$\frac{2}{3}\, a\, ({}^{(1)}E_{\alpha\beta})\, (\bar D_{\sigma 4} \bar D^{\sigma 4} {}^{(1)}V) + \frac{2}{3}\, a\, \bar h_{\alpha\beta}\, ({}^{(1)}\psi)\, (\bar D_{\sigma 5} \bar D^{\sigma 5} {}^{(1)}V) - \frac{1}{6}\, a\, \bar h_{\alpha\beta}\, (\bar D_{\sigma 6} \bar D^{\sigma 6} {}^{(2)}V) \Big)$$

```
In[88]:= org[DD - EE]
```

Out[88]= 0

Stress - Energy Tensor

```
In[89]:= DefTensor[Tmunu[-μ, -ν], M]
         (*Tmunu[μ_,ν_]:=(ρ[u][]+P[u][])u[μ]u[ν]+P[u][]g[μ,ν]*)
```

** DefTensor: Defining tensor Tmunu[-μ, -ν].

```
In[90]:= $Dust = False;
         IndexSet[Tmunu[α_, β_],
          ((ρ[u][] + If[$Dust, 0, P[u][]]) u[α] u[β] + (If[$Dust, 0, P[u][]]) g[α, β])]
```

Out[91]= $\bar g^{\alpha\beta}\, P + u^\alpha\, u^\beta\, (P + \rho)$

```
In[92]:= Tmunu[-α, -β]
```

Out[92]= $\bar g_{\alpha\beta}\, P + u_\alpha\, u_\beta\, (P + \rho)$

```
In[93]:= MyToxPand[CD[-μ]@Tmunu[μ, ν], "NewtonGauge", order];
         ExtractComponents[ExtractOrder[% ah[]^2, 0], h]
         ExtractComponents[ExtractOrder[%% ah[]^2, 1], h]
         ExtractComponents[ExtractOrder[%%% ah[]^2, 2], h]
```

The Splitting of $\frac{4\,P\,u^{\mu}\;u^{\nu}\;(\bar\nabla_{\mu}a)}{(a)^{3}}$ +... was performed in 18.803081 seconds.

Out[94]=

n	$3\mathcal{H}P + 3\mathcal{H}\rho + \rho'$
h	0

Out[95]=

n	$3\mathcal{H}({}^{(1)}P) + 3\mathcal{H}({}^{(1)}\rho) + {}^{(1)}\rho' - 6\mathcal{H}P({}^{(1)}\Phi) - 6\mathcal{H}\rho({}^{(1)}\Phi) - 2\rho'({}^{(1)}\Phi) - 3P({}^{(1)}\psi') - 3\rho({}^{(1)}\psi') + P(\bar{D}_{\mu}\bar{D}^{\mu}\,{}^{(1)}V) + \rho(\bar{D}_{\mu}\bar{D}^{\mu}\,{}^{(1)}V)$
h	$({}^{(1)}B'^{\nu})P + ({}^{(1)}B^{\nu})\mathcal{H}P + ({}^{(1)}B^{\nu})P' + 4\mathcal{H}P({}^{(1)}V^{\nu}) + P'({}^{(1)}V^{\nu}) + P({}^{(1)}V'^{\nu}) + ({}^{(1)}B'^{\nu})\rho + ({}^{(1)}B^{\nu})\mathcal{H}\rho + 4\mathcal{H}({}^{(1)}V^{\nu})\rho + ({}^{(1)}V'^{\nu})\rho + ({}^{(1)}V^{\nu})\rho' + \bar{D}^{\nu}\,{}^{(1)}P + 4\mathcal{H}P(\bar{D}^{\nu}\,{}^{(1)}V) + P'(\bar{D}^{\nu}\,{}^{(1)}V) + 4\mathcal{H}\rho(\bar{D}^{\nu}\,{}^{(1)}V) + \rho'(\bar{D}^{\nu}\,{}^{(1)}V) + P(\bar{D}^{\nu}\,{}^{(1)}V') + \rho(\bar{D}^{\nu}\,{}^{(1)}V') + P(\bar{D}^{\nu}\,{}^{(1)}\Phi) + \rho(\bar{D}^{\nu}\,{}^{(1)}\Phi)$

n

$$2({}^{(1)}B^{\alpha})({}^{(1)}B'_{\alpha})P - 2({}^{(1)}E^{\alpha\mu})({}^{(1)}E'_{\alpha\mu})P + ({}^{(1)}B_{\alpha})({}^{(1)}B^{\alpha})\mathcal{H}P +$$

$$({}^{(1)}B_{\alpha})({}^{(1)}B^{\alpha})P' + \tfrac{3}{2}\mathcal{H}({}^{(2)}P) + 2({}^{(1)}B'^{\alpha})P({}^{(1)}V_{\alpha}) + 8({}^{(1)}B^{\alpha})\mathcal{H}P({}^{(1)}V_{\alpha}) +$$

$$2({}^{(1)}B^{\alpha})P'({}^{(1)}V_{\alpha}) + 4\mathcal{H}P({}^{(1)}V_{\alpha})({}^{(1)}V^{\alpha}) + P'({}^{(1)}V_{\alpha})({}^{(1)}V^{\alpha}) +$$

$$2({}^{(1)}B^{\alpha})P({}^{(1)}V'_{\alpha}) + 2P({}^{(1)}V^{\alpha})({}^{(1)}V'_{\alpha}) + 2({}^{(1)}B^{\alpha})({}^{(1)}B'_{\alpha})\rho -$$

$$2({}^{(1)}E^{\alpha\mu})({}^{(1)}E'_{\alpha\mu})\rho + ({}^{(1)}B_{\alpha})({}^{(1)}B^{\alpha})\mathcal{H}\rho + 2({}^{(1)}B'^{\alpha})({}^{(1)}V_{\alpha})\rho +$$

$$8({}^{(1)}B^{\alpha})\mathcal{H}({}^{(1)}V_{\alpha})\rho + 4\mathcal{H}({}^{(1)}V_{\alpha})({}^{(1)}V^{\alpha})\rho + 2({}^{(1)}B^{\alpha})({}^{(1)}V'_{\alpha})\rho +$$

$$2({}^{(1)}V^{\alpha})({}^{(1)}V'_{\alpha})\rho + 2({}^{(1)}B^{\alpha})({}^{(1)}V_{\alpha})\rho' + ({}^{(1)}V_{\alpha})({}^{(1)}V^{\alpha})\rho' + \tfrac{3}{2}\mathcal{H}({}^{(2)}\rho) +$$

$$\tfrac{1}{2}({}^{(2)}\rho') - 6\mathcal{H}({}^{(1)}P)({}^{(1)}\Phi) - 6\mathcal{H}({}^{(1)}\rho)({}^{(1)}\Phi) - 2({}^{(1)}\rho')({}^{(1)}\Phi) + 12\mathcal{H}P({}^{(1)}\Phi)^{2} +$$

$$12\mathcal{H}\rho({}^{(1)}\Phi)^{2} + 4\rho'({}^{(1)}\Phi)^{2} - 3\mathcal{H}P({}^{(2)}\Phi) - 3\mathcal{H}\rho({}^{(2)}\Phi) - \rho'({}^{(2)}\Phi) - 3({}^{(1)}P)({}^{(1)}\psi') -$$

$$3({}^{(1)}\rho)({}^{(1)}\psi') + 6P({}^{(1)}\Phi)({}^{(1)}\psi') + 6\rho({}^{(1)}\Phi)({}^{(1)}\psi') - 6P({}^{(1)}\psi)({}^{(1)}\psi') -$$

$$6\rho({}^{(1)}\psi)({}^{(1)}\psi') - \tfrac{3}{2}P({}^{(2)}\psi') - \tfrac{3}{2}\rho({}^{(2)}\psi') + ({}^{(1)}B^{\alpha})(\bar{D}_{\alpha}\,{}^{(1)}P) + ({}^{(1)}V^{\alpha})(\bar{D}_{\alpha}\,{}^{(1)}P) +$$

$$2({}^{(1)}B'^{\alpha})P(\bar{D}_{\alpha}\,{}^{(1)}V) + 8({}^{(1)}B^{\alpha})\mathcal{H}P(\bar{D}_{\alpha}\,{}^{(1)}V) + 2({}^{(1)}B^{\alpha})P'(\bar{D}_{\alpha}\,{}^{(1)}V) +$$

$$8\mathcal{H}P({}^{(1)}V^{\alpha})(\bar{D}_{\alpha}\,{}^{(1)}V) + 2P'({}^{(1)}V^{\alpha})(\bar{D}_{\alpha}\,{}^{(1)}V) + 2P({}^{(1)}V'^{\alpha})(\bar{D}_{\alpha}\,{}^{(1)}V) +$$

[illegible]

Out[96]=

$$2\,({}^{(1)}B)\,\rho\,(\bar{D}_\alpha\,{}^{(1)}V)+8\,({}^{(1)}B)\,\mathcal{H}\,\rho\,(\bar{D}_\alpha\,{}^{(1)}V)+8\,\mathcal{H}\,({}^{(1)}V)\,\rho\,(\bar{D}_\alpha\,{}^{(1)}V)+$$

$$2\,({}^{(1)}V'^{\alpha})\,\rho\,(\bar{D}_\alpha\,{}^{(1)}V)+2\,({}^{(1)}B^{\alpha})\,\rho'\,(\bar{D}_\alpha\,{}^{(1)}V)+2\,({}^{(1)}V^{\alpha})\,\rho'\,(\bar{D}_\alpha\,{}^{(1)}V)+$$

$$2\,({}^{(1)}B^{\alpha})\,P\,(\bar{D}_\alpha\,{}^{(1)}V')+2\,P\,({}^{(1)}V^{\alpha})\,(\bar{D}_\alpha\,{}^{(1)}V')+2\,({}^{(1)}B^{\alpha})\,\rho\,(\bar{D}_\alpha\,{}^{(1)}V')+$$

$$2\,({}^{(1)}V^{\alpha})\,\rho\,(\bar{D}_\alpha\,{}^{(1)}V')+({}^{(1)}V^{\alpha})\,(\bar{D}_\alpha\,{}^{(1)}\rho)+({}^{(1)}B^{\alpha})\,P\,(\bar{D}_\alpha\,{}^{(1)}\Phi)+2\,P\,({}^{(1)}V^{\alpha})\,(\bar{D}_\alpha\,{}^{(1)}\Phi)+$$

$$({}^{(1)}B^{\alpha})\,\rho\,(\bar{D}_\alpha\,{}^{(1)}\Phi)+2\,({}^{(1)}V^{\alpha})\,\rho\,(\bar{D}_\alpha\,{}^{(1)}\Phi)-3\,P\,({}^{(1)}V^{\alpha})\,(\bar{D}_\alpha\,{}^{(1)}\psi)-$$

$$3\,({}^{(1)}V^{\alpha})\,\rho\,(\bar{D}_\alpha\,{}^{(1)}\psi)+({}^{(1)}P)\,(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}V)+({}^{(1)}\rho)\,(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}V)-P\,({}^{(1)}\Phi)\,(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}V)-$$

$$\rho\,({}^{(1)}\Phi)\,(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(1)}V)+\frac{1}{2}\,P\,(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(2)}V)+\frac{1}{2}\,\rho\,(\bar{D}_\alpha\,\bar{D}^\alpha\,{}^{(2)}V)+(\bar{D}_\alpha\,{}^{(1)}V)\,(\bar{D}^\alpha\,{}^{(1)}P)+$$

$$4\,\mathcal{H}\,P\,(\bar{D}_\alpha\,{}^{(1)}V)\,(\bar{D}^\alpha\,{}^{(1)}V)+P'\,(\bar{D}_\alpha\,{}^{(1)}V)\,(\bar{D}^\alpha\,{}^{(1)}V)+4\,\mathcal{H}\,\rho\,(\bar{D}_\alpha\,{}^{(1)}V)\,(\bar{D}^\alpha\,{}^{(1)}V)+$$

$$\rho'\,(\bar{D}_\alpha\,{}^{(1)}V)\,(\bar{D}^\alpha\,{}^{(1)}V)+2\,P\,(\bar{D}_\alpha\,{}^{(1)}V')\,(\bar{D}^\alpha\,{}^{(1)}V)+2\,\rho\,(\bar{D}_\alpha\,{}^{(1)}V')\,(\bar{D}^\alpha\,{}^{(1)}V)+(\bar{D}_\alpha\,{}^{(1)}\rho)\,(\bar{D}^\alpha\,{}^{(1)}V)+$$

$$2\,P\,(\bar{D}_\alpha\,{}^{(1)}\Phi)\,(\bar{D}^\alpha\,{}^{(1)}V)+2\,\rho\,(\bar{D}_\alpha\,{}^{(1)}\Phi)\,(\bar{D}^\alpha\,{}^{(1)}V)-3\,P\,(\bar{D}_\alpha\,{}^{(1)}\psi)\,(\bar{D}^\alpha\,{}^{(1)}V)-3\,\rho\,(\bar{D}_\alpha\,{}^{(1)}\psi)\,(\bar{D}^\alpha\,{}^{(1)}V)$$

h

$$\frac{1}{2}\,({}^{(2)}B'^{\nu})\,P-2\,({}^{(1)}B'^{\mu})\,({}^{(1)}E^{\nu}{}_{\mu})\,P+\frac{1}{2}\,({}^{(2)}B^{\nu})\,\mathcal{H}\,P-2\,({}^{(1)}B^{\mu})\,({}^{(1)}E^{\nu}{}_{\mu})\,\mathcal{H}\,P+$$

$$\frac{1}{2}\,({}^{(2)}B^{\nu})\,P'-2\,({}^{(1)}B^{\mu})\,({}^{(1)}E^{\nu}{}_{\mu})\,P'+({}^{(1)}B'^{\nu})\,({}^{(1)}P)+({}^{(1)}B^{\nu})\,\mathcal{H}\,({}^{(1)}P)+$$

$$({}^{(1)}B^{\nu})\,({}^{(1)}P')+2\,({}^{(1)}E'^{\nu}{}_{\mu})\,P\,({}^{(1)}V^{\mu})+4\,\mathcal{H}\,({}^{(1)}P)\,({}^{(1)}V^{\nu})+({}^{(1)}P')\,({}^{(1)}V^{\nu})+$$

$$({}^{(1)}P)\,({}^{(1)}V'^{\nu})+2\,\mathcal{H}\,P\,({}^{(2)}V^{\nu})+\frac{1}{2}\,P'\,({}^{(2)}V^{\nu})+\frac{1}{2}\,P\,({}^{(2)}V'^{\nu})+\frac{1}{2}\,({}^{(2)}B'^{\nu})\,\rho-$$

$$2\,({}^{(1)}B'^{\mu})\,({}^{(1)}E^{\nu}{}_{\mu})\,\rho+\frac{1}{2}\,({}^{(2)}B^{\nu})\,\mathcal{H}\,\rho-2\,({}^{(1)}B^{\mu})\,({}^{(1)}E^{\nu}{}_{\mu})\,\mathcal{H}\,\rho+$$

$$2\,({}^{(1)}E'^{\nu}{}_{\mu})\,({}^{(1)}V^{\mu})\,\rho+2\,\mathcal{H}\,({}^{(2)}V^{\nu})\,\rho+\frac{1}{2}\,({}^{(2)}V'^{\nu})\,\rho+\frac{1}{2}\,({}^{(2)}V^{\nu})\,\rho'+({}^{(1)}B'^{\nu})\,({}^{(1)}\rho)+$$

$$({}^{(1)}B^{\nu})\,\mathcal{H}\,({}^{(1)}\rho)+4\,\mathcal{H}\,({}^{(1)}V^{\nu})\,({}^{(1)}\rho)+({}^{(1)}V'^{\nu})\,({}^{(1)}\rho)+({}^{(1)}V^{\nu})\,({}^{(1)}\rho')-$$

$$2\,({}^{(1)}B'^{\nu})\,P\,({}^{(1)}\Phi)-2\,({}^{(1)}B^{\nu})\,\mathcal{H}\,P\,({}^{(1)}\Phi)-2\,({}^{(1)}B^{\nu})\,P'\,({}^{(1)}\Phi)-4\,\mathcal{H}\,P\,({}^{(1)}V^{\nu})\,({}^{(1)}\Phi)-$$

$$P'\,({}^{(1)}V^{\nu})\,({}^{(1)}\Phi)-P\,({}^{(1)}V'^{\nu})\,({}^{(1)}\Phi)-2\,({}^{(1)}B'^{\nu})\,\rho\,({}^{(1)}\Phi)-2\,({}^{(1)}B^{\nu})\,\mathcal{H}\,\rho\,({}^{(1)}\Phi)-$$

$$4\,\mathcal{H}\,({}^{(1)}V^{\nu})\,\rho\,({}^{(1)}\Phi)-({}^{(1)}V'^{\nu})\,\rho\,({}^{(1)}\Phi)-({}^{(1)}V^{\nu})\,\rho'\,({}^{(1)}\Phi)-({}^{(1)}B^{\nu})\,P\,({}^{(1)}\Phi')-$$

$$({}^{(1)}B^{\nu})\,\rho\,({}^{(1)}\Phi')+2\,({}^{(1)}B'^{\nu})\,P\,({}^{(1)}\psi)+2\,({}^{(1)}B^{\nu})\,\mathcal{H}\,P\,({}^{(1)}\psi)+2\,({}^{(1)}B^{\nu})\,P'\,({}^{(1)}\psi)+$$

$$2\,({}^{(1)}B'^{\nu})\,\rho\,({}^{(1)}\psi)+2\,({}^{(1)}B^{\nu})\,\mathcal{H}\,\rho\,({}^{(1)}\psi)-5\,P\,({}^{(1)}V^{\nu})\,({}^{(1)}\psi')-5\,({}^{(1)}V^{\nu})\,\rho\,({}^{(1)}\psi')+$$

$$P\,({}^{(1)}V^{\mu})\,(\bar{D}_\mu\,{}^{(1)}B^{\nu})+({}^{(1)}V^{\mu})\,\rho\,(\bar{D}_\mu\,{}^{(1)}B^{\nu})-2\,({}^{(1)}E^{\nu\mu})\,(\bar{D}_\mu\,{}^{(1)}P)+$$

$$2\,({}^{(1)}E'^{\nu\mu})\,P\,(\bar{D}_\mu\,{}^{(1)}V)+2\,({}^{(1)}E'^{\nu\mu})\,\rho\,(\bar{D}_\mu\,{}^{(1)}V)+P\,({}^{(1)}V^{\mu})\,(\bar{D}_\mu\,{}^{(1)}V^{\nu})+$$

$$
\begin{aligned}
&\left({}^{(1)}V^{\mu}\right)\rho\left(\bar{D}_{\mu}\,{}^{(1)}V^{\nu}\right)-2\left({}^{(1)}E^{\nu\mu}\right)P\left(\bar{D}_{\mu}\,{}^{(1)}\Phi\right)-2\left({}^{(1)}E^{\nu\mu}\right)\rho\left(\bar{D}_{\mu}\,{}^{(1)}\Phi\right)+\\
&P\left({}^{(1)}V^{\nu}\right)\left(\bar{D}_{\mu}\bar{D}^{\mu}\,{}^{(1)}V\right)+\left({}^{(1)}V^{\nu}\right)\rho\left(\bar{D}_{\mu}\bar{D}^{\mu}\,{}^{(1)}V\right)+P\left({}^{(1)}V^{\mu}\right)\left(\bar{D}_{\mu}\bar{D}^{\nu}\,{}^{(1)}V\right)+\\
&\left({}^{(1)}V^{\mu}\right)\rho\left(\bar{D}_{\mu}\bar{D}^{\nu}\,{}^{(1)}V\right)+P\left(\bar{D}_{\mu}\,{}^{(1)}V\right)\left(\bar{D}^{\mu}\,{}^{(1)}B^{\nu}\right)+\rho\left(\bar{D}_{\mu}\,{}^{(1)}V\right)\left(\bar{D}^{\mu}\,{}^{(1)}B^{\nu}\right)+\\
&P\left(\bar{D}_{\mu}\,{}^{(1)}V^{\nu}\right)\left(\bar{D}^{\mu}\,{}^{(1)}V\right)+\rho\left(\bar{D}_{\mu}\,{}^{(1)}V^{\nu}\right)\left(\bar{D}^{\mu}\,{}^{(1)}V\right)+P\left(\bar{D}_{\mu}\bar{D}^{\nu}\,{}^{(1)}V\right)\left(\bar{D}^{\mu}\,{}^{(1)}V\right)+\\
&\rho\left(\bar{D}_{\mu}\bar{D}^{\nu}\,{}^{(1)}V\right)\left(\bar{D}^{\mu}\,{}^{(1)}V\right)-P\left({}^{(1)}V^{\mu}\right)\left(\bar{D}^{\nu}\,{}^{(1)}B_{\mu}\right)-\left({}^{(1)}V^{\mu}\right)\rho\left(\bar{D}^{\nu}\,{}^{(1)}B_{\mu}\right)-\\
&P\left(\bar{D}_{\mu}\,{}^{(1)}V\right)\left(\bar{D}^{\nu}\,{}^{(1)}B^{\mu}\right)-\rho\left(\bar{D}_{\mu}\,{}^{(1)}V\right)\left(\bar{D}^{\nu}\,{}^{(1)}B^{\mu}\right)+2\left({}^{(1)}\psi\right)\left(\bar{D}^{\nu}\,{}^{(1)}P\right)+\frac{1}{2}\left(\bar{D}^{\nu}\,{}^{(2)}P\right)+\\
&4\mathcal{H}\left({}^{(1)}P\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)+\left({}^{(1)}P'\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)+4\mathcal{H}\left({}^{(1)}\rho\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)+\left({}^{(1)}\rho'\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)-\\
&4\mathcal{H}P\left({}^{(1)}\Phi\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)-P'\left({}^{(1)}\Phi\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)-4\mathcal{H}\rho\left({}^{(1)}\Phi\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)-\rho'\left({}^{(1)}\Phi\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)-\\
&5P\left({}^{(1)}\psi'\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)-5\rho\left({}^{(1)}\psi'\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)+P\left(\bar{D}_{\mu}\bar{D}^{\mu}\,{}^{(1)}V\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)+\\
&\rho\left(\bar{D}_{\mu}\bar{D}^{\mu}\,{}^{(1)}V\right)\left(\bar{D}^{\nu}\,{}^{(1)}V\right)+\left({}^{(1)}P\right)\left(\bar{D}^{\nu}\,{}^{(1)}V'\right)+\left({}^{(1)}\rho\right)\left(\bar{D}^{\nu}\,{}^{(1)}V'\right)-P\left({}^{(1)}\Phi\right)\left(\bar{D}^{\nu}\,{}^{(1)}V'\right)-\\
&\rho\left({}^{(1)}\Phi\right)\left(\bar{D}^{\nu}\,{}^{(1)}V'\right)+2\mathcal{H}P\left(\bar{D}^{\nu}\,{}^{(2)}V\right)+\frac{1}{2}P'\left(\bar{D}^{\nu}\,{}^{(2)}V\right)+2\mathcal{H}\rho\left(\bar{D}^{\nu}\,{}^{(2)}V\right)+\frac{1}{2}\rho'\left(\bar{D}^{\nu}\,{}^{(2)}V\right)+\\
&\frac{1}{2}P\left(\bar{D}^{\nu}\,{}^{(2)}V'\right)+\frac{1}{2}\rho\left(\bar{D}^{\nu}\,{}^{(2)}V'\right)+\left({}^{(1)}P\right)\left(\bar{D}^{\nu}\,{}^{(1)}\Phi\right)+\left({}^{(1)}\rho\right)\left(\bar{D}^{\nu}\,{}^{(1)}\Phi\right)-2P\left({}^{(1)}\Phi\right)\left(\bar{D}^{\nu}\,{}^{(1)}\Phi\right)-\\
&2\rho\left({}^{(1)}\Phi\right)\left(\bar{D}^{\nu}\,{}^{(1)}\Phi\right)+2P\left({}^{(1)}\psi\right)\left(\bar{D}^{\nu}\,{}^{(1)}\Phi\right)+2\rho\left({}^{(1)}\psi\right)\left(\bar{D}^{\nu}\,{}^{(1)}\Phi\right)+\frac{1}{2}P\left(\bar{D}^{\nu}\,{}^{(2)}\Phi\right)+\frac{1}{2}\rho\left(\bar{D}^{\nu}\,{}^{(2)}\Phi\right)
\end{aligned}
$$

```
In[97]:= order = 1

Out[97]= 1
```

In a matter dominated universe, we set the $Dust Boolean to` True'

```
In[98]:= $Dust = True;
         IndexSetDelayed[Tmunu[α_, β_],
          ((ρ[u][] + If[$Dust, 0, P[u][]]) u[α] u[β] + (If[$Dust, 0, P[u][]]) g[α, β])]

In[100]:=
      $FirstOrderVectorPerturbations = False;
      $FirstOrderTensorPerturbations = False;
```

In[102]:=

```
MyToxPand[CD[-μ]@Tmunu[μ, ν], "NewtonGauge", 2];
ExtractComponents[ExtractOrder[% ah[]^2, 0], h]
ExtractComponents[ExtractOrder[%% ah[]^2, 1], h]
ExtractComponents[ExtractOrder[%%% ah[]^2, 2], h]
```

The Splitting of $\frac{4\, u^{\mu}\, u^{\nu}\, \rho\, (\bar{\nabla}_{\mu} a)}{(a)^{3}}$ +... was performed in 5.337081 seconds.

Out[103]=

n	$3\,\mathcal{H}\,\rho + \rho'$
h	0

Out[104]=

n	$3\,\mathcal{H}\,({}^{(1)}\rho) + {}^{(1)}\rho' - 6\,\mathcal{H}\,\rho\,({}^{(1)}\Phi) - 2\,\rho'\,({}^{(1)}\Phi) - 3\,\rho\,({}^{(1)}\psi') + \rho\,(\bar{D}_{\mu}\,\bar{D}^{\mu}\,{}^{(1)}V)$
h	$4\,\mathcal{H}\,\rho\,(\bar{D}^{\nu}\,{}^{(1)}V) + \rho'\,(\bar{D}^{\nu}\,{}^{(1)}V) + \rho\,(\bar{D}^{\nu}\,{}^{(1)}V') + \rho\,(\bar{D}^{\nu}\,{}^{(1)}\Phi)$

Out[105]=

n	$\frac{3}{2}\,\mathcal{H}\,({}^{(2)}\rho) + \frac{1}{2}\,({}^{(2)}\rho') - 6\,\mathcal{H}\,({}^{(1)}\rho)\,({}^{(1)}\Phi) - 2\,({}^{(1)}\rho')\,({}^{(1)}\Phi) + 12\,\mathcal{H}\,\rho\,({}^{(1)}\Phi)^{2} + 4\,\rho'\,({}^{(1)}\Phi)^{2} - 3\,\mathcal{H}\,\rho\,({}^{(2)}\Phi) - \rho'\,({}^{(2)}\Phi) - 3\,({}^{(1)}\rho)\,({}^{(1)}\psi') + 6\,\rho\,({}^{(1)}\Phi)\,({}^{(1)}\psi') - 6\,\rho\,({}^{(1)}\psi)\,({}^{(1)}\psi') - \frac{3}{2}\,\rho\,({}^{(2)}\psi') + ({}^{(1)}\rho)\,(\bar{D}_{\mu}\,\bar{D}^{\mu}\,{}^{(1)}V) - \rho\,({}^{(1)}\Phi)\,(\bar{D}_{\mu}\,\bar{D}^{\mu}\,{}^{(1)}V) + \frac{1}{2}\,\rho\,(\bar{D}_{\mu}\,\bar{D}^{\mu}\,{}^{(2)}V) + 4\,\mathcal{H}\,\rho\,(\bar{D}_{\mu}\,{}^{(1)}V)\,(\bar{D}^{\mu}\,{}^{(1)}V) + \rho'\,(\bar{D}_{\mu}\,{}^{(1)}V)\,(\bar{D}^{\mu}\,{}^{(1)}V) + 2\,\rho\,(\bar{D}_{\mu}\,{}^{(1)}V')\,(\bar{D}^{\mu}\,{}^{(1)}V) + (\bar{D}_{\mu}\,{}^{(1)}\rho)\,(\bar{D}^{\mu}\,{}^{(1)}V) + 2\,\rho\,(\bar{D}_{\mu}\,{}^{(1)}\Phi)\,(\bar{D}^{\mu}\,{}^{(1)}V) - 3\,\rho\,(\bar{D}_{\mu}\,{}^{(1)}\psi)\,(\bar{D}^{\mu}\,{}^{(1)}V)$
h	$\frac{1}{2}\,({}^{(2)}B'^{\nu})\,\rho + \frac{1}{2}\,({}^{(2)}B^{\nu})\,\mathcal{H}\,\rho + 2\,\mathcal{H}\,({}^{(2)}V^{\nu})\,\rho + \frac{1}{2}\,({}^{(2)}V'^{\nu})\,\rho + \frac{1}{2}\,({}^{(2)}V^{\nu})\,\rho' + \rho\,(\bar{D}_{\mu}\,\bar{D}^{\nu}\,{}^{(1)}V)\,(\bar{D}^{\mu}\,{}^{(1)}V) + 4\,\mathcal{H}\,({}^{(1)}\rho)\,(\bar{D}^{\nu}\,{}^{(1)}V) + ({}^{(1)}\rho')\,(\bar{D}^{\nu}\,{}^{(1)}V) - 4\,\mathcal{H}\,\rho\,({}^{(1)}\Phi)\,(\bar{D}^{\nu}\,{}^{(1)}V) - \rho'\,({}^{(1)}\Phi)\,(\bar{D}^{\nu}\,{}^{(1)}V) - 5\,\rho\,({}^{(1)}\psi')\,(\bar{D}^{\nu}\,{}^{(1)}V) + \rho\,(\bar{D}_{\mu}\,\bar{D}^{\mu}\,{}^{(1)}V)\,(\bar{D}^{\nu}\,{}^{(1)}V) + ({}^{(1)}\rho)\,(\bar{D}^{\nu}\,{}^{(1)}V') - \rho\,({}^{(1)}\Phi)\,(\bar{D}^{\nu}\,{}^{(1)}V') + 2\,\mathcal{H}\,\rho\,(\bar{D}^{\nu}\,{}^{(2)}V) + \frac{1}{2}\,\rho'\,(\bar{D}^{\nu}\,{}^{(2)}V) + \frac{1}{2}\,\rho\,(\bar{D}^{\nu}\,{}^{(2)}V') + ({}^{(1)}\rho)\,(\bar{D}^{\nu}\,{}^{(1)}\Phi) - 2\,\rho\,({}^{(1)}\Phi)\,(\bar{D}^{\nu}\,{}^{(1)}\Phi) + 2\,\rho\,({}^{(1)}\psi)\,(\bar{D}^{\nu}\,{}^{(1)}\Phi) + \frac{1}{2}\,\rho\,(\bar{D}^{\nu}\,{}^{(2)}\Phi)$

We can evaluate the Klein-Gordon equation in any gauge up to any order in perturbation theory

In[106]:=

```
Clear[V]
DefScalarFunction[V]
ConformalWeight[V'] = 0;
```

** DefScalarFunction: Defining scalar function V.

In[109]:=

```
MyToxPand[CD[-μ]@CD[μ][φ[]] + V'[φ[]], "AnyGauge", order]
```

The Splitting of $\frac{\bar{\nabla}_\mu \bar{\nabla}^\mu \varphi}{(a)^2}$ +... was performed in 0.492319 seconds.

Out[109]=

$$-\frac{2\mathcal{H}\,\varphi'}{a^2}-\frac{\varphi''}{a^2}+V'[\varphi]+\epsilon\left(-\frac{2\mathcal{H}\left({}^{(1)}\varphi'\right)}{a^2}-\frac{{}^{(1)}\varphi''}{a^2}+\frac{4\mathcal{H}\,\varphi'\left({}^{(1)}\Phi\right)}{a^2}+\frac{2\varphi''\left({}^{(1)}\Phi\right)}{a^2}+\frac{\varphi'\left({}^{(1)}\Phi'\right)}{a^2}+\frac{3\varphi'\left({}^{(1)}\psi'\right)}{a^2}+\frac{\varphi'\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}B\right)}{a^2}-\frac{\varphi'\left(\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}E'\right)}{a^2}+\frac{\bar{D}_\alpha\bar{D}^\alpha\,{}^{(1)}\varphi}{a^2}+\left({}^{(1)}\varphi\right)V''[\varphi]\right)$$

In[110]:=

```
$FirstOrderVectorPerturbations = True;
$FirstOrderTensorPerturbations = True;
```

■ Gauge Transformation

Gauge transformation fields, split in 3+1 on the background. The spatial part is then split into Scalar and Vector parts.

In[112]:=

```
DefTensor[ξ[LI[ord], α], M]
```

** DefTensor: Defining tensor ξ[LI[ord], α].

The gauge transformation to go in the Newtonian gauge is

In[113]:=

```
RulesGaugeField =
 {ξ[LI[ord_], α_] -> (Bsh[LI[ord], LI[0]] - Esh[LI[ord], LI[1]]) n[α] -
    Boole@$FirstOrderVectorPerturbations * Evh[LI[ord], LI[0], α] -
    cd[α][Esh[LI[ord], LI[0]]]}
```

Out[113]=

$$\left\{\xi^{ord\_\underline{\alpha}}\to -E^{ord0\alpha}+\left(B^{ord0}-E^{ord1}\right)\bar{n}^{\alpha}-\bar{D}^{\alpha}E^{ord0}\right\}$$

In[114]:=

```
ξ[LI[1], α] /. RulesGaugeField
```

Out[114]=

$$-\left({}^{(1)}E^{\alpha}\right)+\left({}^{(1)}B-{}^{(1)}E'\right)\bar{n}^{\alpha}-\bar{D}^{\alpha}\,{}^{(1)}E$$

If we use the GaugeChange function of xPert

In[115]:=
```
?GaugeChange
```

GaugeChange[pert, gen] returns the perturbative expression pert changed to a new gauge. The change of gauge is parametrized by the family of vector fields gen[LI[order], a]. Background objects are left untouched.

In[116]:=
```
GaugeChange[dg[LI[1], -α, -β], ξ]
```

Out[116]=

$$\delta g^{1}{}_{\alpha\beta} + \mathcal{L}_{\xi^1} \bar{g}_{\alpha\beta}$$

We see that we need to split the result after specifying what the perturbation of the metric is.

This is done by SplitGaugeChange

In[117]:=
```
?SplitGaugeChange
```

SplitGaugeChange[expr,ListPairs,ξ,h,order] performs a gauge transformation of order 'order' on 'expr', with a conformal transformation of conformal factor 'a[h]', and splits the result according to the background slicing associated with the induced metric 'h'.

'ξ' is the general vector field used for the gauge transformation. It must have a Label–Index (LI) for the order of perturbation (i.e. it has to be of the form ξ[LI[order],index]). 'ListPairs' is the list of rules used for the background slicing.

The theory and formulas for the power expansion of gauge transformation was developed by Bruni et al. in: Class. Quantum Grav. 14, 2585 (1997).

In[118]:=
```
org@SplitGaugeChange[g[-α, -β],
  Join[RulesGaugeField, Rulesmetricmatter["AnyGauge", order]], ξ, h, 1]
```

The Splitting of $\epsilon\, \delta g^{1}{}_{\alpha\beta}\ (a)^2$ +... was performed in 0.525790 seconds.

Out[118]=

$$a^2\, \bar{h}_{\alpha\beta} - a^2\, \bar{n}_\alpha\, \bar{n}_\beta + \epsilon \Big(2 a^2 \left({}^{(1)}E_{\alpha\beta}\right) + 2 a^2 \left({}^{(1)}B\right) \bar{h}_{\alpha\beta}\, \mathcal{H} - 2 a^2 \left({}^{(1)}E'\right) \bar{h}_{\alpha\beta}\, \mathcal{H} - a^2 \left({}^{(1)}B_\beta\right) \bar{n}_\alpha + a^2 \left({}^{(1)}E'_\beta\right) \bar{n}_\alpha - a^2 \left({}^{(1)}B_\alpha\right) \bar{n}_\beta + a^2 \left({}^{(1)}E'_\alpha\right) \bar{n}_\beta - 2 a^2 \left({}^{(1)}B'\right) \bar{n}_\alpha\, \bar{n}_\beta + 2 a^2 \left({}^{(1)}E''\right) \bar{n}_\alpha\, \bar{n}_\beta - 2 a^2 \left({}^{(1)}B\right) \mathcal{H}\, \bar{n}_\alpha\, \bar{n}_\beta + 2 a^2 \left({}^{(1)}E'\right) \mathcal{H}\, \bar{n}_\alpha\, \bar{n}_\beta - 2 a^2\, \bar{n}_\alpha\, \bar{n}_\beta \left({}^{(1)}\Phi\right) - 2 a^2\, \bar{h}_{\alpha\beta} \left({}^{(1)}\psi\right)\Big)$$

Indeed, we can check that after the gauge transformation, the result is in the Newtonian gauge

In[119]:=
```
ExtractComponents[% / (-ah[]^2), h]
```

Out[119]=

	n	h
n	$1 + \epsilon \left(2 \left({}^{(1)}B'\right) - 2 \left({}^{(1)}E''\right) + 2 \left({}^{(1)}B\right) \mathcal{H} - 2 \left({}^{(1)}E'\right) \mathcal{H} + 2 \left({}^{(1)}\Phi\right)\right)$	$\epsilon \left(-\left({}^{(1)}B_\beta\right) + {}^{(1)}E'_\beta\right)$
h	$\epsilon \left(-\left({}^{(1)}B_\alpha\right) + {}^{(1)}E'_\alpha\right)$	$-\bar{h}_{\alpha\beta} + \epsilon \left(-2 \left({}^{(1)}E_{\alpha\beta}\right) - 2 \left({}^{(1)}B\right) \bar{h}_{\alpha\beta}\, \mathcal{H} + 2 \left({}^{(1)}E'\right) \bar{h}_{\alpha\beta}\, \mathcal{H} + 2\, \bar{h}_{\alpha\beta} \left({}^{(1)}\psi\right)\right)$

Or we can check a general transformation. For this we can even use a function which does everything in a simple manner for metric and matter fields perturbations:

In[120]:=

```
?SplitFieldsAndGaugeChange
```

SplitFieldsAndGaugeChange[expr,g,dg,uf,duf,h,order] performs a gauge transformation on expr where expr must involve metric perturbations (or curvature tensor associated with this metric) or matter field perturbation such as the fluid velocity, and splits the results according to the most general gauge choice.
h is the induced metric, and refers to the background n+1 splitting. g is the metric and dg is the metric perturbation. uf is the fluid velocity and duf is the fluid velocity perturbation (when index is up).
ξ is the general vector field used for the gauge transformation which must have a label index for the order of perturbation (it is of the form ξ[LI[order],Index]), and order is the order of the gauge transformation.
The theory and formulas for the power expansion of changes of gauge was developed by Bruni et al. in reference Class. Quantum Grav. 14, 2585 (1997).

In[121]:=

```
MyGaugeChange[expr_, h_, order_] :=
 SplitFieldsAndGaugeChange[expr, g, dg, u, du, h, order]
```

In[122]:=

```
MyGaugeChange[g[α, β], h, order]
```

** DefTensor: Defining tensor ξh[LI[xAct`xPand`Private`ord$136044], σ$136045].

The Splitting of $-\frac{\epsilon\,\delta g^{1\alpha\beta}}{(a)^2}$ +... was performed in 0.663414 seconds.

Out[122]=

$$\frac{\bar h^{\alpha\beta}}{a^2} - \frac{\bar n^\alpha \bar n^\beta}{a^2} + \epsilon\Bigg(-\frac{2\left({}^{(1)}E^{\alpha\beta}\right)}{a^2} + \frac{\left({}^{(1)}B^\beta\right)\bar n^\alpha}{a^2} + \frac{\left({}^{(1)}L'^\beta\right)\bar n^\alpha}{a^2} + \frac{\left({}^{(1)}B^\alpha\right)\bar n^\beta}{a^2} + \frac{\left({}^{(1)}L'^\alpha\right)\bar n^\beta}{a^2} - \frac{2\bar h^{\alpha\beta}\mathcal{H}\left({}^{(1)}T\right)}{a^2} + \frac{2\mathcal H \bar n^\alpha \bar n^\beta \left({}^{(1)}T\right)}{a^2} + \frac{2\bar n^\alpha\bar n^\beta\left({}^{(1)}T'\right)}{a^2} + \frac{2\bar n^\alpha \bar n^\beta\left({}^{(1)}\Phi\right)}{a^2} + \frac{2\bar h^{\alpha\beta}\left({}^{(1)}\psi\right)}{a^2} + \frac{\bar n^\beta\left(\bar D^\alpha {}^{(1)}B\right)}{a^2} - \frac{\bar D^\alpha {}^{(1)}E^\beta}{a^2} + \frac{\bar n^\beta\left(\bar D^\alpha {}^{(1)}L'\right)}{a^2} - \frac{\bar D^\alpha {}^{(1)}L^\beta}{a^2} - \frac{\bar n^\beta\left(\bar D^\alpha {}^{(1)}T\right)}{a^2} + \frac{\bar n^\alpha\left(\bar D^\beta {}^{(1)}B\right)}{a^2} - \frac{\bar D^\beta {}^{(1)}E^\alpha}{a^2} + \frac{\bar n^\alpha\left(\bar D^\beta {}^{(1)}L'\right)}{a^2} - \frac{\bar D^\beta {}^{(1)}L^\alpha}{a^2} - \frac{\bar n^\alpha\left(\bar D^\beta {}^{(1)}T\right)}{a^2} - \frac{2\left(\bar D^\beta \bar D^\alpha {}^{(1)}E\right)}{a^2} - \frac{2\left(\bar D^\beta\bar D^\alpha {}^{(1)}L\right)}{a^2}\Bigg)$$

In[123]:=

```
ExtractComponents[ExtractOrder[%, 1] / (-2 ah[]^2), h]
```

Out[123]=

	n	h
n	$-\frac{\mathcal H\left({}^{(1)}T\right)}{a^4} - \frac{{}^{(1)}T'}{a^4} - \frac{{}^{(1)}\Phi}{a^4}$	$-\frac{{}^{(1)}B^\beta}{2a^4} - \frac{{}^{(1)}L'^\beta}{2a^4} - \frac{\bar D^\beta\,{}^{(1)}B}{2a^4} - \frac{\bar D^\beta\,{}^{(1)}L'}{2a^4} + \frac{\bar D^\beta\,{}^{(1)}T}{2a^4}$
h	$-\frac{{}^{(1)}B^\alpha}{2a^4} - \frac{{}^{(1)}L'^\alpha}{2a^4} - \frac{\bar D^\alpha\,{}^{(1)}B}{2a^4} - \frac{\bar D^\alpha\,{}^{(1)}L'}{2a^4} + \frac{\bar D^\alpha\,{}^{(1)}T}{2a^4}$	$\frac{{}^{(1)}E^{\alpha\beta}}{a^4} + \frac{\bar h^{\alpha\beta}\mathcal H\left({}^{(1)}T\right)}{a^4} - \frac{\bar h^{\alpha\beta}\left({}^{(1)}\psi\right)}{a^4} + \frac{\bar D^\alpha\,{}^{(1)}E^\beta}{2a^4} + \frac{\bar D^\alpha\,{}^{(1)}L^\beta}{2a^4} + \frac{\bar D^\beta\,{}^{(1)}E^\alpha}{2a^4} + \frac{\bar D^\beta\,{}^{(1)}L^\alpha}{2a^4} + \frac{\bar D^\beta\bar D^\alpha\,{}^{(1)}E}{a^4} + \frac{\bar D^\beta\bar D^\alpha\,{}^{(1)}L}{a^4}$

■ Einstein Equations

We may now put together, the curvature perturbation and the matter perturbation to reproduce some of the well-known equations of cosmological perturbation theory on a flat FLRW background space-time.

In[124]:=
```
DefConstantSymbol[κ]
DefConstantSymbol[Λ]
```

```
** DefConstantSymbol: Defining constant symbol κ.
```

```
** DefConstantSymbol: Defining constant symbol Λ.
```

In[126]:=
```
MyGR[μ_, ν_] := EinsteinCD[μ, ν] + g[μ, ν] Λ / κ^(2) - κ^(2) * Tmunu[μ, ν]

MyGR[μ, ν]
```

Out[127]=

$$G[\overline{\nabla}]^{\mu\nu} + \frac{\Lambda\, \overline{g}^{\mu\nu}}{\kappa^2} - \kappa^2\, u^{\mu}\, u^{\nu}\, \rho$$

In[128]:=

```
MyGRresult = MyToxPand[MyGR[μ, -ν], "NewtonGauge", 1]
```

The Splitting of $\frac{\Lambda\,\delta^{\mu}{}_{\nu}}{\kappa^2}$ +... was performed in 3.447331 seconds.

Out[128]=

$$\frac{\Lambda\,\delta^{\mu}{}_{\nu}}{\kappa^2}-\frac{\delta^{\mu}{}_{\nu}\,\mathcal{H}^2}{a^2}-\frac{2\,\delta^{\mu}{}_{\nu}\,\mathcal{H}'}{a^2}+\frac{2\,\mathcal{H}^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}}{a^2}-\frac{2\,\mathcal{H}'\,\bar{n}^{\mu}\,\bar{n}_{\nu}}{a^2}-\kappa^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}\,\rho+$$

$$\epsilon\left(\frac{{}^{(1)}E^{\mu}{}_{\nu}{}''}{a^2}+\frac{2\left({}^{(1)}E^{\mu}{}_{\nu}{}'\right)\mathcal{H}}{a^2}-\frac{2\left({}^{(1)}B^{\mu}\right)\mathcal{H}^2\,\bar{n}_{\nu}}{a^2}+\frac{2\left({}^{(1)}B^{\mu}\right)\mathcal{H}'\,\bar{n}_{\nu}}{a^2}-\kappa^2\left({}^{(1)}B_{\nu}\right)\bar{n}^{\mu}\,\rho-\right.$$

$$\kappa^2\,\bar{n}_{\nu}\left({}^{(1)}V^{\mu}\right)\rho-\kappa^2\,\bar{n}^{\mu}\left({}^{(1)}V_{\nu}\right)\rho-\kappa^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\rho\right)+\frac{2\,\delta^{\mu}{}_{\nu}\,\mathcal{H}^2\left({}^{(1)}\Phi\right)}{a^2}+$$

$$\frac{4\,\delta^{\mu}{}_{\nu}\,\mathcal{H}'\left({}^{(1)}\Phi\right)}{a^2}-\frac{4\,\mathcal{H}^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\Phi\right)}{a^2}+\frac{4\,\mathcal{H}'\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\Phi\right)}{a^2}+\frac{2\,\delta^{\mu}{}_{\nu}\,\mathcal{H}\left({}^{(1)}\Phi'\right)}{a^2}+$$

$$\frac{2\,\mathcal{H}\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\Phi'\right)}{a^2}+\frac{6\,\delta^{\mu}{}_{\nu}\,\mathcal{H}\left({}^{(1)}\psi'\right)}{a^2}-\frac{2\,\bar{h}^{\mu}{}_{\nu}\,\mathcal{H}\left({}^{(1)}\psi'\right)}{a^2}+\frac{3\,\delta^{\mu}{}_{\nu}\left({}^{(1)}\psi''\right)}{a^2}-$$

$$\frac{\bar{h}^{\mu}{}_{\nu}\left({}^{(1)}\psi''\right)}{a^2}+\frac{3\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\psi''\right)}{a^2}-\frac{\bar{n}_{\nu}\left(\bar{D}_{\alpha}\,{}^{(1)}E^{\mu\alpha}{}'\right)}{a^2}+\frac{\bar{n}_{\nu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}B^{\mu}\right)}{2\,a^2}+$$

$$\frac{\bar{n}^{\mu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}B_{\nu}\right)}{2\,a^2}-\frac{\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}E^{\mu}{}_{\nu}}{a^2}+\frac{\delta^{\mu}{}_{\nu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}\Phi\right)}{a^2}+\frac{\bar{n}^{\mu}\,\bar{n}_{\nu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}\Phi\right)}{a^2}-$$

$$\frac{2\,\delta^{\mu}{}_{\nu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}\psi\right)}{a^2}+\frac{\bar{h}^{\mu}{}_{\nu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}\psi\right)}{a^2}-\frac{\mathcal{H}\left(\bar{D}^{\mu}\,{}^{(1)}B_{\nu}\right)}{a^2}-\frac{\bar{D}^{\mu}\,{}^{(1)}B_{\nu}{}'}{2\,a^2}-$$

$$\kappa^2\,\bar{n}_{\nu}\,\rho\left(\bar{D}^{\mu}\,{}^{(1)}V\right)-\frac{2\,\mathcal{H}\,\bar{n}_{\nu}\left(\bar{D}^{\mu}\,{}^{(1)}\Phi\right)}{a^2}-\frac{2\,\bar{n}_{\nu}\left(\bar{D}^{\mu}\,{}^{(1)}\psi'\right)}{a^2}-\frac{\mathcal{H}\left(\bar{D}_{\nu}\,{}^{(1)}B^{\mu}\right)}{a^2}-\frac{\bar{D}_{\nu}\,{}^{(1)}B^{\mu}{}'}{2\,a^2}-$$

$$\left.\kappa^2\,\bar{n}^{\mu}\,\rho\left(\bar{D}_{\nu}\,{}^{(1)}V\right)-\frac{2\,\mathcal{H}\,\bar{n}^{\mu}\left(\bar{D}_{\nu}\,{}^{(1)}\Phi\right)}{a^2}-\frac{2\,\bar{n}^{\mu}\left(\bar{D}_{\nu}\,{}^{(1)}\psi'\right)}{a^2}-\frac{\bar{D}_{\nu}\,\bar{D}^{\mu}\,{}^{(1)}\Phi}{a^2}+\frac{\bar{D}_{\nu}\,\bar{D}^{\mu}\,{}^{(1)}\psi}{a^2}\right)$$

Here is the background part

In[129]:=
```
ExtractComponents[ExtractOrder[MyGRresult, 0] ah[]^2, h, {"Time", "Time"}]
ExtractComponents[ExtractOrder[MyGRresult, 0] ah[]^2, h, {"Space", "Space"}]

Hdot = IndexSolve[Evaluate[% h[-μ, ν]] == 0, Hh[LI[0], LI[1]]]
```

Out[129]=

$$\frac{\Lambda a^2}{\kappa^2} - 3\,\mathcal{H}^2 + \kappa^2 a^2 \rho$$

Out[130]=

$$\frac{\Lambda a^2 \, \bar{h}^{\mu}{}_{\nu}}{\kappa^2} - \bar{h}^{\mu}{}_{\nu}\, \mathcal{H}^2 - 2\, \bar{h}^{\mu}{}_{\nu}\, \mathcal{H}'$$

Out[131]=

$$\left\{\text{HoldPattern}\left[\mathcal{H}'\right] :\to \text{Module}\left[\{\},\ \frac{\Lambda\,(a)^2}{2\,\kappa^2} - \frac{(\mathcal{H})^2}{2}\right]\right\}$$

In[132]:=
```
ExtractComponents[ExtractOrder[MyGRresult, 1] ah[]^4, h]
```

Out[132]=

	n	h
n	$\kappa^2 a^4 \left({}^{(1)}\rho\right) + 6\, a^2\, \mathcal{H}^2 \left({}^{(1)}\Phi\right) + 6\, a^2\, \mathcal{H} \left({}^{(1)}\psi'\right) - 2\, a^2 \left(\bar{D}_{\alpha}\, \bar{D}^{\alpha}\, {}^{(1)}\psi\right)$	$-\kappa^2 a^4 \left({}^{(1)}B_{\nu}\right) \rho - \kappa^2 a^4 \left({}^{(1)}V_{\nu}\right) \rho + \frac{1}{2}\, a^2 \left(\bar{D}_{\alpha}\, \bar{D}^{\alpha}\, {}^{(1)}B_{\nu}\right) - \kappa^2 a^4 \rho \left(\bar{D}_{\nu}\, {}^{(1)}V\right) - 2\, a^2\, \mathcal{H} \left(\bar{D}_{\nu}\, {}^{(1)}\Phi\right) - 2\, a^2 \left(\bar{D}_{\nu}\, {}^{(1)}\psi'\right)$
h	$2\, a^2 \left({}^{(1)}B^{\mu}\right) \mathcal{H}^2 - 2\, a^2 \left({}^{(1)}B^{\mu}\right) \mathcal{H}' + \kappa^2 a^4 \left({}^{(1)}V^{\mu}\right) \rho + a^2 \left(\bar{D}_{\alpha}\, {}^{(1)}E'^{\mu\alpha}\right) - \frac{1}{2}\, a^2 \left(\bar{D}_{\alpha}\, \bar{D}^{\alpha}\, {}^{(1)}B^{\mu}\right) + \kappa^2 a^4 \rho \left(\bar{D}^{\mu}\, {}^{(1)}V\right) + 2\, a^2\, \mathcal{H} \left(\bar{D}^{\mu}\, {}^{(1)}\Phi\right) + 2\, a^2 \left(\bar{D}^{\mu}\, {}^{(1)}\psi'\right)$	$a^2 \left({}^{(1)}E''^{\mu}{}_{\nu}\right) + 2\, a^2 \left({}^{(1)}E'^{\mu}{}_{\nu}\right) \mathcal{H} + 2\, a^2\, \bar{h}^{\mu}{}_{\nu}\, \mathcal{H}^2 \left({}^{(1)}\Phi\right) + 4\, a^2\, \bar{h}^{\mu}{}_{\nu}\, \mathcal{H}' \left({}^{(1)}\Phi\right) + 2\, a^2\, \bar{h}^{\mu}{}_{\nu}\, \mathcal{H} \left({}^{(1)}\Phi'\right) + 4\, a^2\, \bar{h}^{\mu}{}_{\nu}\, \mathcal{H} \left({}^{(1)}\psi'\right) + 2\, a^2\, \bar{h}^{\mu}{}_{\nu} \left({}^{(1)}\psi''\right) - a^2 \left(\bar{D}_{\alpha}\, \bar{D}^{\alpha}\, {}^{(1)}E^{\mu}{}_{\nu}\right) + a^2\, \bar{h}^{\mu}{}_{\nu} \left(\bar{D}_{\alpha}\, \bar{D}^{\alpha}\, {}^{(1)}\Phi\right) - a^2\, \bar{h}^{\mu}{}_{\nu} \left(\bar{D}_{\alpha}\, \bar{D}^{\alpha}\, {}^{(1)}\psi\right) - a^2\, \mathcal{H} \left(\bar{D}^{\mu}\, {}^{(1)}B_{\nu}\right) - \frac{1}{2}\, a^2 \left(\bar{D}^{\mu}\, {}^{(1)}B'_{\nu}\right) - a^2\, \mathcal{H} \left(\bar{D}_{\nu}\, {}^{(1)}B^{\mu}\right) - \frac{1}{2}\, a^2 \left(\bar{D}_{\nu}\, {}^{(1)}B'^{\mu}\right) - a^2 \left(\bar{D}_{\nu}\, \bar{D}^{\mu}\, {}^{(1)}\Phi\right) + a^2 \left(\bar{D}_{\nu}\, \bar{D}^{\mu}\, {}^{(1)}\psi\right)$

In[133]:=
```
STFPart[Projectorh[MyToxPand[MyGR[-β, -α], "NewtonGauge", 1]], h] // org
```

The Splitting of $R[\overline{\nabla}]_{\alpha\beta}$ +... was performed in 3.444247 seconds.

Out[133]=

$$\epsilon\left(\frac{2\Lambda a^2\left({}^{(1)}E_{\alpha\beta}\right)}{\kappa^2}+{}^{(1)}E''_{\alpha\beta}+2\left({}^{(1)}E'_{\alpha\beta}\right)\mathcal{H}-2\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}^2-4\left({}^{(1)}E_{\alpha\beta}\right)\mathcal{H}'-\mathcal{H}\left(\overline{D}_\alpha\,{}^{(1)}B_\beta\right)-\frac{1}{2}\left(\overline{D}_\alpha\,{}^{(1)}B'_\beta\right)-\mathcal{H}\left(\overline{D}_\beta\,{}^{(1)}B_\alpha\right)-\frac{1}{2}\left(\overline{D}_\beta\,{}^{(1)}B'_\alpha\right)-\overline{D}_\beta\overline{D}_\alpha\,{}^{(1)}\Phi+\overline{D}_\beta\overline{D}_\alpha\,{}^{(1)}\psi-\overline{D}_\gamma\overline{D}^\gamma\,{}^{(1)}E_{\alpha\beta}+\frac{1}{3}\bar{h}_{\alpha\beta}\left(\overline{D}_\gamma\overline{D}^\gamma\,{}^{(1)}\Phi\right)-\frac{1}{3}\bar{h}_{\alpha\beta}\left(\overline{D}_\gamma\overline{D}^\gamma\,{}^{(1)}\psi\right)\right)$$

In[134]:=

We could set vectors and tensors to zero at first order

In[135]:=
```
$FirstOrderVectorPerturbations = False;
$FirstOrderTensorPerturbations = False;
```

For perfect fluid $\phi = \psi$, the constraint could be implemented automatical

In[137]:=
```
PerfectFluidConstraint = MakeRule[{ψh[LI[1], LI[0]], ϕh[LI[1], LI[0]]}]
AutomaticRules[ψh, PerfectFluidConstraint]
```

Out[137]=

$\{\text{HoldPattern}[{}^{(1)}\psi] :\to \text{Module}[\{\}, {}^{(1)}\Phi]\}$

Rules {1} have been declared as DownValues for ψh.

In[139]:=
```
MyGRresult2 = MyToxPand[MyGR[μ, -ν], "NewtonGauge", 1]
```

The Splitting of $\frac{\Lambda\,\delta^{\mu}{}_{\nu}}{\kappa^2}$ +... was performed in 2.476473 seconds.

Out[139]=

$$\frac{\Lambda\,\delta^{\mu}{}_{\nu}}{\kappa^2}-\frac{\delta^{\mu}{}_{\nu}\,\mathcal{H}^2}{a^2}-\frac{2\,\delta^{\mu}{}_{\nu}\,\mathcal{H}'}{a^2}+\frac{2\,\mathcal{H}^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}}{a^2}-\frac{2\,\mathcal{H}'\,\bar{n}^{\mu}\,\bar{n}_{\nu}}{a^2}-\kappa^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}\,\rho+\epsilon\left(-\kappa^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\rho\right)+\frac{2\,\delta^{\mu}{}_{\nu}\,\mathcal{H}^2\left({}^{(1)}\Phi\right)}{a^2}+\frac{4\,\delta^{\mu}{}_{\nu}\,\mathcal{H}'\left({}^{(1)}\Phi\right)}{a^2}-\frac{4\,\mathcal{H}^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\Phi\right)}{a^2}+\frac{4\,\mathcal{H}'\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\Phi\right)}{a^2}+\frac{8\,\delta^{\mu}{}_{\nu}\,\mathcal{H}\left({}^{(1)}\Phi'\right)}{a^2}-\frac{2\,\bar{h}^{\mu}{}_{\nu}\,\mathcal{H}\left({}^{(1)}\Phi'\right)}{a^2}+\frac{2\,\mathcal{H}\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\Phi'\right)}{a^2}+\frac{3\,\delta^{\mu}{}_{\nu}\left({}^{(1)}\Phi''\right)}{a^2}-\frac{\bar{h}^{\mu}{}_{\nu}\left({}^{(1)}\Phi''\right)}{a^2}+\frac{3\,\bar{n}^{\mu}\,\bar{n}_{\nu}\left({}^{(1)}\Phi''\right)}{a^2}-\frac{\delta^{\mu}{}_{\nu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}\Phi\right)}{a^2}+\frac{\bar{h}^{\mu}{}_{\nu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}\Phi\right)}{a^2}+\frac{\bar{n}^{\mu}\,\bar{n}_{\nu}\left(\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}\Phi\right)}{a^2}-\kappa^2\,\bar{n}_{\nu}\,\rho\left(\bar{D}^{\mu}\,{}^{(1)}V\right)-\frac{2\,\mathcal{H}\,\bar{n}_{\nu}\left(\bar{D}^{\mu}\,{}^{(1)}\Phi\right)}{a^2}-\frac{2\,\bar{n}_{\nu}\left(\bar{D}^{\mu}\,{}^{(1)}\Phi'\right)}{a^2}-\kappa^2\,\bar{n}^{\mu}\,\rho\left(\bar{D}_{\nu}\,{}^{(1)}V\right)-\frac{2\,\mathcal{H}\,\bar{n}^{\mu}\left(\bar{D}_{\nu}\,{}^{(1)}\Phi\right)}{a^2}-\frac{2\,\bar{n}^{\mu}\left(\bar{D}_{\nu}\,{}^{(1)}\Phi'\right)}{a^2}\right)$$

Here are familar equations

In[140]:=
```
ExtractComponents[
 ExtractOrder[MyGRresult2 / 2 /. Hdot, 1] ah[]^2, h, {"Time", "Time"}]
```

Out[140]=

$$\frac{1}{2}\kappa^2\,a^2\left({}^{(1)}\rho\right)+3\,\mathcal{H}^2\left({}^{(1)}\Phi\right)+3\,\mathcal{H}\left({}^{(1)}\Phi'\right)-\bar{D}_{\alpha}\,\bar{D}^{\alpha}\,{}^{(1)}\Phi$$

In[141]:=
```
ExtractComponents[
 ExtractOrder[MyGRresult2 / 2 /. Hdot, 1] ah[]^2, h, {"Time", "Space"}]
```

Out[141]=

$$-\frac{1}{2}\kappa^2\,a^2\,\rho\left(\bar{D}_{\nu}\,{}^{(1)}V\right)-\mathcal{H}\left(\bar{D}_{\nu}\,{}^{(1)}\Phi\right)-\bar{D}_{\nu}\,{}^{(1)}\Phi'$$

In[142]:=
```
ExtractComponents[
  ExtractOrder[MyGRresult2 /. Hdot, 1] ah[]^2, h, {"Space", "Space"}];
% * h[-μ, ν] / 6 // org
```

Out[143]=

$$\frac{\Lambda\,a^2\left({}^{(1)}\Phi\right)}{\kappa^2}+3\,\mathcal{H}\left({}^{(1)}\Phi'\right)+{}^{(1)}\Phi''$$

```
In[144]:=
    $FirstOrderVectorPerturbations = True;
    $FirstOrderTensorPerturbations = True;
```

Bianchi space-times

There is no special need to define a new manifold with a new background metric.

In order to specify that we now want to work with a Bianchi space-time, we only say that the background splitting we use has different properties.

That is, it should be defined with a different normal vector, which implies a different induced metric, for which the extrinsic curvature is different.

The constants of structures of this new slicing should also be different.

In order to achieve this new slicing, the only command which is needed is SetSlicing.

We call it with the same background metric g, but with a different normal vector (nI) and induced metric (hI),. Of course, the SpaceType which is specified as an argument is now set to BIanchiI.

```
In[146]:=
    SetSlicing[g, nI, hI, cdI, {"|", "D̄"}, "BianchiI"]

    ** DefTensor: Defining tensor nI[σ$157207].
    ** DefTensor: Defining symmetric metric tensor hI[-σ$157207, -σ$157208].
    ** DefTensor: Defining antisymmetric tensor epsilonhI[-α, -β, -γ].
    ** DefTensor: Defining tensor TetrahI[-α, -β, -γ, -λ].
    ** DefTensor: Defining tensor TetrahI†[-α, -β, -γ, -λ].
    ** DefCovD: Defining covariant derivative cdI[-σ$157207].
    ** DefTensor: Defining vanishing torsion tensor TorsioncdI[α, -β, -γ].
    ** DefTensor: Defining
     symmetric Christoffel tensor ChristoffelcdI[α, -β, -γ].
    ** DefTensor: Defining Riemann tensor RiemanncdI[-α, -β, -γ, -λ].
    ** DefTensor: Defining symmetric Ricci tensor RiccicdI[-α, -β].
    ** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
    ** DefTensor: Defining Ricci scalar RicciScalarcdI[].
    ** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.
    ** DefTensor: Defining symmetric Einstein tensor EinsteincdI[-α, -β].
    ** DefTensor: Defining Weyl tensor WeylcdI[-α, -β, -γ, -λ].
    ** DefTensor: Defining symmetric TFRicci tensor TFRiccicdI[-α, -β].
    ** DefTensor: Defining Kretschmann scalar KretschmanncdI[].
    ** DefCovD:  Computing RiemannToWeylRules for dim 4
    ** DefCovD:  Computing RicciToTFRicci for dim 4
```

```
** DefCovD:  Computing RicciToEinsteinRules for dim 4

** DefTensor: Defining weight +2 density DethI[]. Determinant.

** DefTensor: Defining extrinsic curvature tensor
 ExtrinsicKhI[α, β]. Associated to vector nI

** DefTensor: Defining acceleration vector
 AccelerationnI[α]. Associated to vector nI

** DefInertHead: Defining projector inert-head ProjectorhI.

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for hI.

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for hI.

   Rules {1, 2} have been declared as UpValues for nI.

   Rules {1, 2, 3, 4} have been declared as UpValues for nI.

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for nI.

** DefTensor: Defining tensor
 ahI[LI[xAct`xPand`Private`p$158475], LI[xAct`xPand`Private`q$158475]].

** DefTensor: Defining tensor
 HhI[LI[xAct`xPand`Private`p$158480], LI[xAct`xPand`Private`q$158480]].

** DefTensor: Defining symmetric metric tensor gahI2[-σ$158486, -σ$158487].

** DefTensor: Defining inverse metric tensor
 InvgahI2[σ$158486, σ$158487]. Metric is frozen!

** DefMetric: Don't know yet how to define epsilon for a frozen metric.

** DefCovD: Defining covariant derivative CDahI2[-σ$158486].

** DefTensor: Defining vanishing torsion tensor TorsionCDahI2[α, -β, -γ].

** DefTensor: Defining
 symmetric Christoffel tensor ChristoffelCDahI2[α, -β, -γ].

** DefTensor: Defining Riemann tensor RiemannDownCDahI2[-α, -β, -γ, -λ].

** DefTensor: Defining Riemann tensor
 RiemannCDahI2[-α, -β, -γ, λ]. Antisymmetric only in the first pair.

** DefTensor: Defining symmetric Ricci tensor RicciCDahI2[-α, -β].

** DefCovD:  Contractions of Riemann automatically replaced by Ricci.

** DefTensor: Defining Ricci scalar RicciScalarCDahI2[].

** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.

** DefTensor: Defining symmetric Einstein tensor EinsteinCDahI2[-α, -β].

** MakeRule: Potential problems moving indices on the LHS.

** DefTensor: Defining Weyl tensor WeylCDahI2[-α, -β, -γ, -λ].

** DefTensor: Defining symmetric TFRicci tensor TFRicciCDahI2[-α, -β].

** DefTensor: Defining Kretschmann scalar KretschmannCDahI2[].

** DefCovD:  Computing RiemannToWeylRules for dim 4

** DefCovD:  Computing RicciToEinsteinRules for dim 4
```

```
** DefTensor: Defining weight +2 density DetgahI2[]. Determinant.

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for g.

   Rules {1} have been declared as UpValues for g.

** DefTensor: Defining tensor ConnectionhI[-σ$157207, -σ$157208, -σ$157209].

** DefTensor: Defining tensor CShI[-σ$157207, -σ$157208, -σ$157209].

** DefTensor: Defining tensor KhI[LI[xAct`xPand`Private`p$159106],
  LI[xAct`xPand`Private`q$159106], -σ$157207, -σ$157208].
```

In[147]:=

```
PrintAs[KhI] ^= "σ";
PrintAs[hI] ^= "h";
PrintAs[nI] ^= "n";
```

We now define the perturbations fields associated with the metric and matter perturbations, for this new background splitting.

In[150]:=

```
DefMetricFields[g, dg, hI]
DefMatterFields[u, du, hI]
```

```
** Warning: Metric perturbation already
  defined. This cannot be redefined without undefining it. **

** DefTensor: Defining tensor
 ϕhI[LI[xAct`xPand`Private`p$159366], LI[xAct`xPand`Private`q$159366]].

** DefTensor: Defining tensor
 BshI[LI[xAct`xPand`Private`p$159371], LI[xAct`xPand`Private`q$159371]].

** DefTensor: Defining tensor
 BvhI[LI[xAct`xPand`Private`p$159376], LI[xAct`xPand`Private`q$159376], -α].

** DefTensor: Defining tensor
 ψhI[LI[xAct`xPand`Private`p$159382], LI[xAct`xPand`Private`q$159382]].

** DefTensor: Defining tensor
 EshI[LI[xAct`xPand`Private`p$159387], LI[xAct`xPand`Private`q$159387]].

** DefTensor: Defining tensor
 EvhI[LI[xAct`xPand`Private`p$159392], LI[xAct`xPand`Private`q$159392], -α].

** DefTensor: Defining tensor EthI[
  LI[xAct`xPand`Private`p$159398], LI[xAct`xPand`Private`q$159398], -α, -β].

** DefTensor: Defining tensor
 ThI[LI[xAct`xPand`Private`p$159407], LI[xAct`xPand`Private`q$159407]].

** DefTensor: Defining tensor
 LshI[LI[xAct`xPand`Private`p$159412], LI[xAct`xPand`Private`q$159412]].

** DefTensor: Defining tensor
 LvhI[LI[xAct`xPand`Private`p$159417], LI[xAct`xPand`Private`q$159417], -α].

** DefProjectedTensor: Projection properties for the tensor φ
  have been defined for another slicing. New projection properties on the
   hypersurfaces associated with the induced metric hI are now added.

** DefProjectedTensor: The tensor φ
  already exists. The projection properties on the hypersurfaces
   associated with the induced metric hI are now defined.
```

```
** DefProjectedTensor: Projection properties for the tensor ρu
  have been defined for another slicing. New projection properties on the
   hypersurfaces associated with the induced metric hI are now added.

** DefProjectedTensor: The tensor ρu
  already exists. The projection properties on the hypersurfaces
   associated with the induced metric hI are now defined.

** DefProjectedTensor: Projection properties for the tensor Pu
  have been defined for another slicing. New projection properties on the
   hypersurfaces associated with the induced metric hI are now added.

** DefProjectedTensor: The tensor Pu
  already exists. The projection properties on the hypersurfaces
   associated with the induced metric hI are now defined.

** DefTensor: Defining tensor VspathIu[
  LI[xAct`xPand`Private`p$159431], LI[xAct`xPand`Private`q$159431], -α].

** DefTensor: Defining tensor
 V0hIu[LI[xAct`xPand`Private`p$159437], LI[xAct`xPand`Private`q$159437]].

** DefTensor: Defining tensor
 VshIu[LI[xAct`xPand`Private`p$159442], LI[xAct`xPand`Private`q$159442]].

** DefTensor: Defining tensor
 VvhIu[LI[xAct`xPand`Private`p$159447], LI[xAct`xPand`Private`q$159447], -α].

** Warning: Tensor u
 is already defined. It cannot be redefined without undefining it. **

** Warning: Tensor du
 is already defined. It cannot be redefined without undefining it. **
```

Perturbations

In[152]:=

```
MyToxPandI[expr_, gauge_, order_] := ToxPand[expr, dg, u, du, hI, gauge, order]
```

In[153]:=

```
MyGR[-μ, -ν]
```

Out[153]=

$$G[\overline{\nabla}]_{\mu\nu} + \frac{\Lambda\, \overline{g}_{\mu\nu}}{\kappa^2} - \kappa^2\, u_\mu\, u_\nu\, \rho$$

In[154]:=

```
MyToxPandI[MyGR[-μ, -ν], "NewtonGauge", 1]
```

```
** DefTensor: Defining tensor ChristoffelCDCDahI2[α, -β, -γ].
```

The Splitting of $R[\overline{\nabla}]_{\mu\nu}$ +... was performed in 14.914102 seconds.

Out[154]=

$$\frac{\Lambda a^2\, h_{\mu\nu}}{\kappa^2} - \mathcal{H}^2\, h_{\mu\nu} - 2\,\mathcal{H}'\, h_{\mu\nu} - \frac{1}{2}\, h_{\mu\nu}\, \sigma_{\alpha\beta}\, \sigma^{\alpha\beta} + 2\,\mathcal{H}\, \sigma_{\mu\nu} - 2\, \sigma_\mu{}^\alpha\, \sigma_{\nu\alpha} +$$
$$\sigma'_{\mu\nu} - \frac{\Lambda a^2\, n_\mu\, n_\nu}{\kappa^2} + 3\,\mathcal{H}^2\, n_\mu\, n_\nu - \frac{1}{2}\, \sigma_{\alpha\beta}\, \sigma^{\alpha\beta}\, n_\mu\, n_\nu - \kappa^2\, a^2\, n_\mu\, n_\nu\, \rho +$$

$$\in \left(\frac{2 \Lambda a^2 \left({}^{(1)}E_{\mu\nu}\right)}{\kappa^2} + {}^{(1)}E''_{\mu\nu} + 2\left({}^{(1)}E'_{\mu\nu}\right)\mathcal{H} - 2\left({}^{(1)}E_{\mu\nu}\right)\mathcal{H}^2 - 4\left({}^{(1)}E_{\mu\nu}\right)\mathcal{H}' - \right.$$

$$\left({}^{(1)}E'_{\alpha\beta}\right) h_{\mu\nu}\, \sigma^{\alpha\beta} - \left({}^{(1)}E_{\mu\nu}\right)\sigma_{\alpha\beta}\,\sigma^{\alpha\beta} + 2\left({}^{(1)}E^{\alpha\beta}\right) h_{\mu\nu}\,\sigma_{\alpha}{}^{\gamma}\,\sigma_{\beta\gamma} -$$

$$2\left({}^{(1)}E'_{\nu\alpha}\right)\sigma_{\mu}{}^{\alpha} - 2\left({}^{(1)}E'_{\mu\alpha}\right)\sigma_{\nu}{}^{\alpha} + 4\left({}^{(1)}E^{\alpha\beta}\right)\sigma_{\mu\alpha}\,\sigma_{\nu\beta} - \frac{\Lambda a^2 \left({}^{(1)}B_{\nu}\right) n_{\mu}}{\kappa^2} +$$

$$\left({}^{(1)}B_{\nu}\right)\mathcal{H}^2 n_{\mu} + 2\left({}^{(1)}B_{\nu}\right)\mathcal{H}' n_{\mu} + \frac{1}{2}\left({}^{(1)}B_{\nu}\right)\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\, n_{\mu} - 2\left({}^{(1)}B^{\alpha}\right)\mathcal{H}\,\sigma_{\nu\alpha}\, n_{\mu} +$$

$$2\left({}^{(1)}B^{\alpha}\right)\sigma_{\alpha\beta}\,\sigma_{\nu}{}^{\beta}\, n_{\mu} - \left({}^{(1)}B^{\alpha}\right)\sigma'_{\nu\alpha}\, n_{\mu} - \frac{\Lambda a^2 \left({}^{(1)}B_{\mu}\right) n_{\nu}}{\kappa^2} + \left({}^{(1)}B_{\mu}\right)\mathcal{H}^2 n_{\nu} +$$

$$2\left({}^{(1)}B_{\mu}\right)\mathcal{H}' n_{\nu} + \frac{1}{2}\left({}^{(1)}B_{\mu}\right)\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\, n_{\nu} - 2\left({}^{(1)}B^{\alpha}\right)\mathcal{H}\,\sigma_{\mu\alpha}\, n_{\nu} +$$

$$2\left({}^{(1)}B^{\alpha}\right)\sigma_{\alpha\beta}\,\sigma_{\mu}{}^{\beta}\, n_{\nu} - \left({}^{(1)}B^{\alpha}\right)\sigma'_{\mu\alpha}\, n_{\nu} - \left({}^{(1)}E'_{\alpha\beta}\right)\sigma^{\alpha\beta}\, n_{\mu}\, n_{\nu} +$$

$$2\left({}^{(1)}E^{\alpha\beta}\right)\sigma_{\alpha}{}^{\gamma}\,\sigma_{\beta\gamma}\, n_{\mu}\, n_{\nu} - \kappa^2 a^2\left({}^{(1)}B_{\nu}\right) n_{\mu}\,\rho - \kappa^2 a^2\left({}^{(1)}B_{\mu}\right) n_{\nu}\,\rho -$$

$$\kappa^2 a^2 n_{\nu}\left({}^{(1)}V_{\mu}\right)\rho - \kappa^2 a^2 n_{\mu}\left({}^{(1)}V_{\nu}\right)\rho - \kappa^2 a^2 n_{\mu}\, n_{\nu}\left({}^{(1)}\rho\right) + 2\mathcal{H}^2 h_{\mu\nu}\left({}^{(1)}\phi\right) +$$

$$4\mathcal{H}' h_{\mu\nu}\left({}^{(1)}\phi\right) + h_{\mu\nu}\,\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\left({}^{(1)}\phi\right) - 4\mathcal{H}\,\sigma_{\mu\nu}\left({}^{(1)}\phi\right) + 4\sigma_{\mu}{}^{\alpha}\,\sigma_{\nu\alpha}\left({}^{(1)}\phi\right) -$$

$$2\sigma'_{\mu\nu}\left({}^{(1)}\phi\right) - \frac{2\Lambda a^2 n_{\mu}\, n_{\nu}\left({}^{(1)}\phi\right)}{\kappa^2} - 2\kappa^2 a^2 n_{\mu}\, n_{\nu}\,\rho\left({}^{(1)}\phi\right) + 2\mathcal{H}\, h_{\mu\nu}\left({}^{(1)}\phi'\right) -$$

$$\sigma_{\mu\nu}\left({}^{(1)}\phi'\right) - \frac{2\Lambda a^2 h_{\mu\nu}\left({}^{(1)}\psi\right)}{\kappa^2} + 2\mathcal{H}^2 h_{\mu\nu}\left({}^{(1)}\psi\right) + 4\mathcal{H}' h_{\mu\nu}\left({}^{(1)}\psi\right) +$$

$$h_{\mu\nu}\,\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\left({}^{(1)}\psi\right) - \frac{\mathcal{H}' h_{\mu\nu}\,\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\left({}^{(1)}\psi\right)}{\mathcal{H}^2} - \frac{2 h_{\mu\nu}\,\sigma_{\alpha}{}^{\gamma}\,\sigma^{\alpha\beta}\,\sigma_{\beta\gamma}\left({}^{(1)}\psi\right)}{\mathcal{H}} -$$

$$\frac{2\Lambda a^2 \sigma_{\mu\nu}\left({}^{(1)}\psi\right)}{\kappa^2 \mathcal{H}} - 2\mathcal{H}\,\sigma_{\mu\nu}\left({}^{(1)}\psi\right) + \frac{6\mathcal{H}'\sigma_{\mu\nu}\left({}^{(1)}\psi\right)}{\mathcal{H}} - \frac{2\mathcal{H}'^2\sigma_{\mu\nu}\left({}^{(1)}\psi\right)}{\mathcal{H}^3} +$$

$$\frac{\mathcal{H}''\sigma_{\mu\nu}\left({}^{(1)}\psi\right)}{\mathcal{H}^2} + \frac{\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\,\sigma_{\mu\nu}\left({}^{(1)}\psi\right)}{\mathcal{H}} + 4\sigma_{\mu}{}^{\alpha}\,\sigma_{\nu\alpha}\left({}^{(1)}\psi\right) - \frac{4\mathcal{H}'\sigma_{\mu}{}^{\alpha}\,\sigma_{\nu\alpha}\left({}^{(1)}\psi\right)}{\mathcal{H}^2} -$$

$$\frac{4\sigma_{\alpha\beta}\,\sigma_{\mu}{}^{\alpha}\,\sigma_{\nu}{}^{\beta}\left({}^{(1)}\psi\right)}{\mathcal{H}} + \frac{h_{\mu\nu}\,\sigma^{\alpha\beta}\,\sigma'_{\alpha\beta}\left({}^{(1)}\psi\right)}{\mathcal{H}} + \frac{2\sigma_{\nu}{}^{\alpha}\,\sigma'_{\mu\alpha}\left({}^{(1)}\psi\right)}{\mathcal{H}} - 4\sigma'_{\mu\nu}\left({}^{(1)}\psi\right) +$$

$$\frac{2\mathcal{H}'\sigma'_{\mu\nu}\left({}^{(1)}\psi\right)}{\mathcal{H}^2} + \frac{2\sigma_{\mu}{}^{\alpha}\,\sigma'_{\nu\alpha}\left({}^{(1)}\psi\right)}{\mathcal{H}} - \frac{\sigma''_{\mu\nu}\left({}^{(1)}\psi\right)}{\mathcal{H}} - \frac{\mathcal{H}'\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\, n_{\mu}\, n_{\nu}\left({}^{(1)}\psi\right)}{\mathcal{H}^2} -$$

$$\frac{2\,\sigma_{\alpha}{}^{\gamma}\,\sigma^{\alpha\beta}\,\sigma_{\beta\gamma}\,n_{\mu}\,n_{\nu}\,\left({}^{(1)}\psi\right)}{\mathcal{H}}+\frac{\sigma^{\alpha\beta}\,\sigma'_{\alpha\beta}\,n_{\mu}\,n_{\nu}\,\left({}^{(1)}\psi\right)}{\mathcal{H}}+4\,\mathcal{H}\,h_{\mu\nu}\,\left({}^{(1)}\psi'\right)+$$

$$\frac{h_{\mu\nu}\,\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\,\left({}^{(1)}\psi'\right)}{\mathcal{H}}-5\,\sigma_{\mu\nu}\,\left({}^{(1)}\psi'\right)+\frac{2\,\mathcal{H}'\,\sigma_{\mu\nu}\,\left({}^{(1)}\psi'\right)}{\mathcal{H}^{2}}+\frac{4\,\sigma_{\mu}{}^{\alpha}\,\sigma_{\nu\alpha}\,\left({}^{(1)}\psi'\right)}{\mathcal{H}}-$$

$$\frac{2\,\sigma'_{\mu\nu}\,\left({}^{(1)}\psi'\right)}{\mathcal{H}}-6\,\mathcal{H}\,n_{\mu}\,n_{\nu}\,\left({}^{(1)}\psi'\right)+\frac{\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\,n_{\mu}\,n_{\nu}\,\left({}^{(1)}\psi'\right)}{\mathcal{H}}+2\,h_{\mu\nu}\,\left({}^{(1)}\psi''\right)-$$

$$\frac{\sigma_{\mu\nu}\,\left({}^{(1)}\psi''\right)}{\mathcal{H}}+\sigma_{\nu}{}^{\alpha}\,\left(\overline{D}_{\alpha}\,{}^{(1)}B_{\mu}\right)+\sigma_{\mu}{}^{\alpha}\,\left(\overline{D}_{\alpha}\,{}^{(1)}B_{\nu}\right)+\sigma_{\nu}{}^{\alpha}\,n_{\mu}\,\left(\overline{D}_{\alpha}\,{}^{(1)}\phi\right)+$$

$$\sigma_{\mu}{}^{\alpha}\,n_{\nu}\,\left(\overline{D}_{\alpha}\,{}^{(1)}\phi\right)+3\,\sigma_{\nu}{}^{\alpha}\,n_{\mu}\,\left(\overline{D}_{\alpha}\,{}^{(1)}\psi\right)-\frac{\mathcal{H}'\,\sigma_{\nu}{}^{\alpha}\,n_{\mu}\,\left(\overline{D}_{\alpha}\,{}^{(1)}\psi\right)}{\mathcal{H}^{2}}+3\,\sigma_{\mu}{}^{\alpha}\,n_{\nu}\,\left(\overline{D}_{\alpha}\,{}^{(1)}\psi\right)-$$

$$\frac{\mathcal{H}'\,\sigma_{\mu}{}^{\alpha}\,n_{\nu}\,\left(\overline{D}_{\alpha}\,{}^{(1)}\psi\right)}{\mathcal{H}^{2}}+\frac{\sigma_{\nu}{}^{\alpha}\,n_{\mu}\,\left(\overline{D}_{\alpha}\,{}^{(1)}\psi'\right)}{\mathcal{H}}+\frac{\sigma_{\mu}{}^{\alpha}\,n_{\nu}\,\left(\overline{D}_{\alpha}\,{}^{(1)}\psi'\right)}{\mathcal{H}}+$$

$$\frac{1}{2}\,n_{\nu}\,\left(\overline{D}_{\alpha}\,\overline{D}^{\alpha}\,{}^{(1)}B_{\mu}\right)+\frac{1}{2}\,n_{\mu}\,\left(\overline{D}_{\alpha}\,\overline{D}^{\alpha}\,{}^{(1)}B_{\nu}\right)-\overline{D}_{\alpha}\,\overline{D}^{\alpha}\,{}^{(1)}E_{\mu\nu}+h_{\mu\nu}\,\left(\overline{D}_{\alpha}\,\overline{D}^{\alpha}\,{}^{(1)}\phi\right)-$$

$$h_{\mu\nu}\,\left(\overline{D}_{\alpha}\,\overline{D}^{\alpha}\,{}^{(1)}\psi\right)+\frac{\sigma_{\mu\nu}\,\left(\overline{D}_{\alpha}\,\overline{D}^{\alpha}\,{}^{(1)}\psi\right)}{\mathcal{H}}+2\,n_{\mu}\,n_{\nu}\,\left(\overline{D}_{\alpha}\,\overline{D}^{\alpha}\,{}^{(1)}\psi\right)-\frac{\sigma_{\nu}{}^{\alpha}\,\left(\overline{D}_{\alpha}\,\overline{D}_{\mu}\,{}^{(1)}\psi\right)}{\mathcal{H}}-$$

$$\frac{\sigma_{\mu}{}^{\alpha}\,\left(\overline{D}_{\alpha}\,\overline{D}_{\nu}\,{}^{(1)}\psi\right)}{\mathcal{H}}+\frac{\sigma'_{\nu\alpha}\,n_{\mu}\,\left(\overline{D}^{\alpha}\,{}^{(1)}\psi\right)}{\mathcal{H}}+\frac{\sigma'_{\mu\alpha}\,n_{\nu}\,\left(\overline{D}^{\alpha}\,{}^{(1)}\psi\right)}{\mathcal{H}}+h_{\mu\nu}\,\sigma^{\alpha\beta}\,\left(\overline{D}_{\beta}\,{}^{(1)}B_{\alpha}\right)+$$

$$\sigma^{\alpha\beta}\,n_{\mu}\,n_{\nu}\,\left(\overline{D}_{\beta}\,{}^{(1)}B_{\alpha}\right)-2\,\sigma^{\alpha\beta}\,n_{\nu}\,\left(\overline{D}_{\beta}\,{}^{(1)}E_{\mu\alpha}\right)-2\,\sigma^{\alpha\beta}\,n_{\mu}\,\left(\overline{D}_{\beta}\,{}^{(1)}E_{\nu\alpha}\right)-$$

$$\frac{2\,\sigma_{\alpha}{}^{\beta}\,\sigma_{\nu}{}^{\alpha}\,n_{\mu}\,\left(\overline{D}_{\beta}\,{}^{(1)}\psi\right)}{\mathcal{H}}-\frac{2\,\sigma_{\alpha}{}^{\beta}\,\sigma_{\mu}{}^{\alpha}\,n_{\nu}\,\left(\overline{D}_{\beta}\,{}^{(1)}\psi\right)}{\mathcal{H}}+\frac{h_{\mu\nu}\,\sigma^{\alpha\beta}\,\left(\overline{D}_{\beta}\,\overline{D}_{\alpha}\,{}^{(1)}\psi\right)}{\mathcal{H}}-$$

$$\frac{\sigma^{\alpha\beta}\,n_{\mu}\,n_{\nu}\,\left(\overline{D}_{\beta}\,\overline{D}_{\alpha}\,{}^{(1)}\psi\right)}{\mathcal{H}}-\mathcal{H}\,\left(\overline{D}_{\mu}\,{}^{(1)}B_{\nu}\right)-\frac{1}{2}\,\left(\overline{D}_{\mu}\,{}^{(1)}B'_{\nu}\right)+\sigma^{\alpha\beta}\,n_{\nu}\,\left(\overline{D}_{\mu}\,{}^{(1)}E_{\alpha\beta}\right)-$$

$$\kappa^{2}\,a^{2}\,n_{\nu}\,\rho\,\left(\overline{D}_{\mu}\,{}^{(1)}V\right)-2\,\mathcal{H}\,n_{\nu}\,\left(\overline{D}_{\mu}\,{}^{(1)}\phi\right)-\frac{\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\,n_{\nu}\,\left(\overline{D}_{\mu}\,{}^{(1)}\psi\right)}{\mathcal{H}}-2\,n_{\nu}\,\left(\overline{D}_{\mu}\,{}^{(1)}\psi'\right)-$$

$$\mathcal{H}\,\left(\overline{D}_{\nu}\,{}^{(1)}B_{\mu}\right)-\frac{1}{2}\,\left(\overline{D}_{\nu}\,{}^{(1)}B'_{\mu}\right)+\sigma^{\alpha\beta}\,n_{\mu}\,\left(\overline{D}_{\nu}\,{}^{(1)}E_{\alpha\beta}\right)-\kappa^{2}\,a^{2}\,n_{\mu}\,\rho\,\left(\overline{D}_{\nu}\,{}^{(1)}V\right)-$$

$$\left.2\,\mathcal{H}\,n_{\mu}\,\left(\overline{D}_{\nu}\,{}^{(1)}\phi\right)-\frac{\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\,n_{\mu}\,\left(\overline{D}_{\nu}\,{}^{(1)}\psi\right)}{\mathcal{H}}-2\,n_{\mu}\,\left(\overline{D}_{\nu}\,{}^{(1)}\psi'\right)-\overline{D}_{\nu}\,\overline{D}_{\mu}\,{}^{(1)}\phi+\overline{D}_{\nu}\,\overline{D}_{\mu}\,{}^{(1)}\psi\right)$$

```
In[155]:=
ExtractComponents[ExtractOrder[%, 0] ahI[]^4, hI]
ExtractComponents[ExtractOrder[%%, 1] ahI[]^4, hI]
```

Out[155]=

	nI	hI
nI	$-\frac{\Lambda a^6}{\kappa^2} + 3 a^4 \mathcal{H}^2 - \frac{1}{2} a^4 \sigma_{\alpha\beta} \sigma^{\alpha\beta} - \kappa^2 a^6 \rho$	0
hI	0	$\frac{\Lambda a^6 h_{\mu\nu}}{\kappa^2} - a^4 \mathcal{H}^2 h_{\mu\nu} - 2 a^4 \mathcal{H}' h_{\mu\nu} - \frac{1}{2} a^4 h_{\mu\nu} \sigma_{\alpha\beta} \sigma^{\alpha\beta} + 2 a^4 \mathcal{H} \sigma_{\mu\nu} - 2 a^4 \sigma_{\mu}{}^{\alpha} \sigma_{\nu\alpha} + a^4 \sigma'_{\mu\nu}$

Out[156]=

	nI	hI
nI	$-a^4 \left({}^{(1)}E'_{\alpha\beta}\right) \sigma^{\alpha\beta} + 2 a^4 \left({}^{(1)}E^{\alpha\beta}\right) \sigma_{\alpha}{}^{\gamma} \sigma_{\beta\gamma} - \kappa^2 a^6 \left({}^{(1)}\rho\right) - \frac{2 \Lambda a^6 \left({}^{(1)}\phi\right)}{\kappa^2} - 2 \kappa^2 a^6 \rho \left({}^{(1)}\phi\right) - \frac{a^4 \mathcal{H}' \sigma_{\alpha\beta} \sigma^{\alpha\beta} \left({}^{(1)}\psi\right)}{\mathcal{H}^2} - \frac{2 a^4 \sigma_{\alpha}{}^{\gamma} \sigma^{\alpha\beta} \sigma_{\beta\gamma} \left({}^{(1)}\psi\right)}{\mathcal{H}} + \frac{a^4 \sigma^{\alpha\beta} \sigma'_{\alpha\beta} \left({}^{(1)}\psi\right)}{\mathcal{H}} - 6 a^4 \mathcal{H} \left({}^{(1)}\psi'\right) + \frac{a^4 \sigma_{\alpha\beta} \sigma^{\alpha\beta} \left({}^{(1)}\psi'\right)}{\mathcal{H}} + 2 a^4 \left(\bar{D}_{\alpha} \bar{D}^{\alpha}\, {}^{(1)}\psi\right) + a^4 \sigma^{\alpha\beta} \left(\bar{D}_{\beta}\, {}^{(1)}B_{\alpha}\right) - \frac{a^4 \sigma^{\alpha\beta} \left(\bar{D}_{\beta} \bar{D}_{\alpha}\, {}^{(1)}\psi\right)}{\mathcal{H}}$	$\frac{\Lambda a^6 \left({}^{(1)}B_{\nu}\right)}{\kappa^2} - a^4 \left({}^{(1)}B_{\nu}\right) \mathcal{H}^2 - 2 a^4 \left({}^{(1)}B_{\nu}\right) \mathcal{H}' - \frac{1}{2} a^4 \left({}^{(1)}B_{\nu}\right) \sigma_{\alpha\beta} \sigma^{\alpha\beta} + 2 a^4 \left({}^{(1)}B^{\alpha}\right) \mathcal{H} \sigma_{\nu\alpha} - 2 a^4 \left({}^{(1)}B^{\alpha}\right) \sigma_{\alpha\beta} \sigma_{\nu}{}^{\beta} + a^4 \left({}^{(1)}B^{\alpha}\right) \sigma'_{\nu\alpha} + \kappa^2 a^6 \left({}^{(1)}B_{\nu}\right) \rho + \kappa^2 a^6 \left({}^{(1)}V_{\nu}\right) \rho - a^4 \sigma_{\nu}{}^{\alpha} \left(\bar{D}_{\alpha}\, {}^{(1)}\phi\right) - 3 a^4 \sigma_{\nu}{}^{\alpha} \left(\bar{D}_{\alpha}\, {}^{(1)}\psi\right) + \frac{a^4 \mathcal{H}' \sigma_{\nu}{}^{\alpha} \left(\bar{D}_{\alpha}\, {}^{(1)}\psi\right)}{\mathcal{H}^2} - \frac{a^4 \sigma_{\nu}{}^{\alpha} \left(\bar{D}_{\alpha}\, {}^{(1)}\psi'\right)}{\mathcal{H}} - \frac{1}{2} a^4 \left(\bar{D}_{\alpha} \bar{D}^{\alpha}\, {}^{(1)}B_{\nu}\right) - \frac{a^4 \sigma'_{\nu\alpha} \left(\bar{D}^{\alpha}\, {}^{(1)}\psi\right)}{\mathcal{H}} + 2 a^4 \sigma^{\alpha\beta} \left(\bar{D}_{\beta}\, {}^{(1)}E_{\nu\alpha}\right) + \frac{2 a^4 \sigma_{\alpha}{}^{\beta} \sigma_{\nu}{}^{\alpha} \left(\bar{D}_{\beta}\, {}^{(1)}\psi\right)}{\mathcal{H}} - a^4 \sigma^{\alpha\beta} \left(\bar{D}_{\nu}\, {}^{(1)}E_{\alpha\beta}\right) + \kappa^2 a^6 \rho \left(\bar{D}_{\nu}\, {}^{(1)}V\right) + 2 a^4 \mathcal{H} \left(\bar{D}_{\nu}\, {}^{(1)}\phi\right) + \frac{a^4 \sigma_{\alpha\beta} \sigma^{\alpha\beta} \left(\bar{D}_{\nu}\, {}^{(1)}\psi\right)}{\mathcal{H}} + 2 a^4 \left(\bar{D}_{\nu}\, {}^{(1)}\psi'\right)$
hI	$\frac{\Lambda a^6 \left({}^{(1)}B_{\mu}\right)}{\kappa^2} - a^4 \left({}^{(1)}B_{\mu}\right) \mathcal{H}^2 - 2 a^4 \left({}^{(1)}B_{\mu}\right) \mathcal{H}' - \frac{1}{2} a^4 \left({}^{(1)}B_{\mu}\right) \sigma_{\alpha\beta} \sigma^{\alpha\beta} + 2 a^4 \left({}^{(1)}B^{\alpha}\right) \mathcal{H} \sigma_{\mu\alpha} - 2 a^4 \left({}^{(1)}B^{\alpha}\right) \sigma_{\alpha\beta} \sigma_{\mu}{}^{\beta} +$	$\frac{2 \Lambda a^6 \left({}^{(1)}E_{\mu\nu}\right)}{\kappa^2} + a^4 \left({}^{(1)}E''_{\mu\nu}\right) + 2 a^4 \left({}^{(1)}E'_{\mu\nu}\right) \mathcal{H} - 2 a^4 \left({}^{(1)}E_{\mu\nu}\right) \mathcal{H}^2 - 4 a^4 \left({}^{(1)}E_{\mu\nu}\right) \mathcal{H}' - a^4 \left({}^{(1)}E'_{\alpha\beta}\right) h_{\mu\nu} \sigma^{\alpha\beta} -$

$$a^4 \left({}^{(1)}B^{\alpha}\right) \sigma'_{\mu\alpha} + \kappa^2 a^6 \left({}^{(1)}B_{\mu}\right) \rho +$$
$$\kappa^2 a^6 \left({}^{(1)}V_{\mu}\right) \rho - a^4 \sigma_{\mu}{}^{\alpha} \left(\bar{D}_{\alpha}\, {}^{(1)}\phi\right) -$$
$$3 a^4 \sigma_{\mu}{}^{\alpha} \left(\bar{D}_{\alpha}\, {}^{(1)}\psi\right) + \frac{a^4 \mathcal{H}' \sigma_{\mu}{}^{\alpha} \left(\bar{D}_{\alpha}\, {}^{(1)}\psi\right)}{\mathcal{H}^2} -$$
$$\frac{a^4 \sigma_{\mu}{}^{\alpha} \left(\bar{D}_{\alpha}\, {}^{(1)}\psi'\right)}{\mathcal{H}} - \frac{1}{2} a^4 \left(\bar{D}_{\alpha} \bar{D}^{\alpha}\, {}^{(1)}B_{\mu}\right) -$$
$$\frac{a^4 \sigma'_{\mu\alpha} \left(\bar{D}^{\alpha}\, {}^{(1)}\psi\right)}{\mathcal{H}} + 2 a^4 \sigma^{\alpha\beta} \left(\bar{D}_{\beta}\, {}^{(1)}E_{\mu\alpha}\right) +$$
$$\frac{2 a^4 \sigma_{\alpha}{}^{\beta} \sigma_{\mu}{}^{\alpha} \left(\bar{D}_{\beta}\, {}^{(1)}\psi\right)}{\mathcal{H}} - a^4 \sigma^{\alpha\beta} \left(\bar{D}_{\mu}\, {}^{(1)}E_{\alpha\beta}\right) +$$
$$\kappa^2 a^6 \rho \left(\bar{D}_{\mu}\, {}^{(1)}V\right) + 2 a^4 \mathcal{H} \left(\bar{D}_{\mu}\, {}^{(1)}\phi\right) +$$
$$\frac{a^4 \sigma_{\alpha\beta} \sigma^{\alpha\beta} \left(\bar{D}_{\mu}\, {}^{(1)}\psi\right)}{\mathcal{H}} + 2 a^4 \left(\bar{D}_{\mu}\, {}^{(1)}\psi'\right)$$

$$a^4 \left({}^{(1)}E_{\mu\nu}\right) \sigma_{\alpha\beta} \sigma^{\alpha\beta} +$$
$$2 a^4 \left({}^{(1)}E^{\alpha\beta}\right) h_{\mu\nu} \sigma_{\alpha}{}^{\gamma} \sigma_{\beta\gamma} -$$
$$2 a^4 \left({}^{(1)}E'_{\nu\alpha}\right) \sigma_{\mu}{}^{\alpha} -$$
$$2 a^4 \left({}^{(1)}E'_{\mu\alpha}\right) \sigma_{\nu}{}^{\alpha} +$$
$$4 a^4 \left({}^{(1)}E^{\alpha\beta}\right) \sigma_{\mu\alpha} \sigma_{\nu\beta} +$$
$$2 a^4 \mathcal{H}^2 h_{\mu\nu} \left({}^{(1)}\phi\right) + 4 a^4 \mathcal{H}' h_{\mu\nu} \left({}^{(1)}\phi\right) +$$
$$a^4 h_{\mu\nu} \sigma_{\alpha\beta} \sigma^{\alpha\beta} \left({}^{(1)}\phi\right) -$$
$$4 a^4 \mathcal{H} \sigma_{\mu\nu} \left({}^{(1)}\phi\right) + 4 a^4 \sigma_{\mu}{}^{\alpha} \sigma_{\nu\alpha} \left({}^{(1)}\phi\right) -$$
$$2 a^4 \sigma'_{\mu\nu} \left({}^{(1)}\phi\right) + 2 a^4 \mathcal{H} h_{\mu\nu} \left({}^{(1)}\phi'\right) -$$
$$a^4 \sigma_{\mu\nu} \left({}^{(1)}\phi'\right) - \frac{2 \Lambda a^6 h_{\mu\nu} \left({}^{(1)}\psi\right)}{\kappa^2} +$$
$$2 a^4 \mathcal{H}^2 h_{\mu\nu} \left({}^{(1)}\psi\right) + 4 a^4 \mathcal{H}' h_{\mu\nu} \left({}^{(1)}\psi\right) +$$
$$a^4 h_{\mu\nu} \sigma_{\alpha\beta} \sigma^{\alpha\beta} \left({}^{(1)}\psi\right) -$$
$$\frac{a^4 \mathcal{H}' h_{\mu\nu} \sigma_{\alpha\beta} \sigma^{\alpha\beta} \left({}^{(1)}\psi\right)}{\mathcal{H}^2} -$$
$$\frac{2 a^4 h_{\mu\nu} \sigma_{\alpha}{}^{\gamma} \sigma^{\alpha\beta} \sigma_{\beta\gamma} \left({}^{(1)}\psi\right)}{\mathcal{H}} -$$
$$\frac{2 \Lambda a^6 \sigma_{\mu\nu} \left({}^{(1)}\psi\right)}{\kappa^2 \mathcal{H}} - 2 a^4 \mathcal{H} \sigma_{\mu\nu} \left({}^{(1)}\psi\right) +$$
$$\frac{6 a^4 \mathcal{H}' \sigma_{\mu\nu} \left({}^{(1)}\psi\right)}{\mathcal{H}} - \frac{2 a^4 \mathcal{H}'^2 \sigma_{\mu\nu} \left({}^{(1)}\psi\right)}{\mathcal{H}^3} +$$
$$\frac{a^4 \mathcal{H}'' \sigma_{\mu\nu} \left({}^{(1)}\psi\right)}{\mathcal{H}^2} + \frac{a^4 \sigma_{\alpha\beta} \sigma^{\alpha\beta} \sigma_{\mu\nu} \left({}^{(1)}\psi\right)}{\mathcal{H}} +$$
$$4 a^4 \sigma_{\mu}{}^{\alpha} \sigma_{\nu\alpha} \left({}^{(1)}\psi\right) -$$
$$\frac{4 a^4 \mathcal{H}' \sigma_{\mu}{}^{\alpha} \sigma_{\nu\alpha} \left({}^{(1)}\psi\right)}{\mathcal{H}^2} - \frac{4 a^4 \sigma_{\alpha\beta} \sigma_{\mu}{}^{\alpha} \sigma_{\nu}{}^{\beta} \left({}^{(1)}\psi\right)}{\mathcal{H}} +$$
$$\frac{a^4 h_{\mu\nu} \sigma^{\alpha\beta} \sigma'_{\alpha\beta} \left({}^{(1)}\psi\right)}{\mathcal{H}} + \frac{2 a^4 \sigma_{\nu}{}^{\alpha} \sigma'_{\mu\alpha} \left({}^{(1)}\psi\right)}{\mathcal{H}} -$$
$$4 a^4 \sigma'_{\mu\nu} \left({}^{(1)}\psi\right) + \frac{2 a^4 \mathcal{H}' \sigma'_{\mu\nu} \left({}^{(1)}\psi\right)}{\mathcal{H}^2} +$$
$$\frac{2 a^4 \sigma_{\mu}{}^{\alpha} \sigma'_{\nu\alpha} \left({}^{(1)}\psi\right)}{\mathcal{H}} - \frac{a^4 \sigma''_{\mu\nu} \left({}^{(1)}\psi\right)}{\mathcal{H}} +$$

$$4 a^4 \mathcal{H}\, h_{\mu\nu} \left({}^{(1)}\psi'\right) + \frac{a \ldots h_{\mu\nu}\, \sigma_{\alpha\beta}\, \sigma \ldots \left(\ldots \psi\right)}{\mathcal{H}} -$$

$$5 a^4 \sigma_{\mu\nu} \left({}^{(1)}\psi'\right) + \frac{2 a^4 \mathcal{H}'\, \sigma_{\mu\nu} \left({}^{(1)}\psi'\right)}{\mathcal{H}^2} +$$

$$\frac{4 a^4 \sigma_{\mu}{}^{\alpha}\, \sigma_{\nu\alpha} \left({}^{(1)}\psi'\right)}{\mathcal{H}} - \frac{2 a^4 \sigma'_{\mu\nu} \left({}^{(1)}\psi'\right)}{\mathcal{H}} +$$

$$2 a^4 h_{\mu\nu} \left({}^{(1)}\psi''\right) - \frac{a^4 \sigma_{\mu\nu} \left({}^{(1)}\psi''\right)}{\mathcal{H}} +$$

$$a^4 \sigma_{\nu}{}^{\alpha} \left(\bar{D}_\alpha\, {}^{(1)}B_\mu\right) + a^4 \sigma_{\mu}{}^{\alpha} \left(\bar{D}_\alpha\, {}^{(1)}B_\nu\right) -$$

$$a^4 \left(\bar{D}_\alpha \bar{D}^\alpha\, {}^{(1)}E_{\mu\nu}\right) + a^4 h_{\mu\nu} \left(\bar{D}_\alpha \bar{D}^\alpha\, {}^{(1)}\phi\right) -$$

$$a^4 h_{\mu\nu} \left(\bar{D}_\alpha \bar{D}^\alpha\, {}^{(1)}\psi\right) + \frac{a^4 \sigma_{\mu\nu} \left(\bar{D}_\alpha \bar{D}^\alpha\, {}^{(1)}\psi\right)}{\mathcal{H}} -$$

$$\frac{a^4 \sigma_{\nu}{}^{\alpha} \left(\bar{D}_\alpha \bar{D}_\mu\, {}^{(1)}\psi\right)}{\mathcal{H}} - \frac{a^4 \sigma_{\mu}{}^{\alpha} \left(\bar{D}_\alpha \bar{D}_\nu\, {}^{(1)}\psi\right)}{\mathcal{H}} +$$

$$a^4 h_{\mu\nu}\, \sigma^{\alpha\beta} \left(\bar{D}_\beta\, {}^{(1)}B_\alpha\right) +$$

$$\frac{a^4 h_{\mu\nu}\, \sigma^{\alpha\beta} \left(\bar{D}_\beta \bar{D}_\alpha\, {}^{(1)}\psi\right)}{\mathcal{H}} -$$

$$a^4 \mathcal{H} \left(\bar{D}_\mu\, {}^{(1)}B_\nu\right) - \frac{1}{2} a^4 \left(\bar{D}_\mu\, {}^{(1)}B'_\nu\right) -$$

$$a^4 \mathcal{H} \left(\bar{D}_\nu\, {}^{(1)}B_\mu\right) - \frac{1}{2} a^4 \left(\bar{D}_\nu\, {}^{(1)}B'_\mu\right) -$$

$$a^4 \left(\bar{D}_\nu \bar{D}_\mu\, {}^{(1)}\phi\right) + a^4 \left(\bar{D}_\nu \bar{D}_\mu\, {}^{(1)}\psi\right)$$

To Be Done

- To Be Done

  Null Geodesic equation

  Boltzman equation

  Null Geodesic Deviation equation

Summary of public functions and variables

```
In[157]:=
?xAct`xPand`*
```

▼ xAct`xPand`

a	Es	SpaceType	VisualizeTens⋱ or	$FirstOrderVe⋱ ctorPerturba⋱ tions
av	Et	SplitFieldsAnd⋱ GaugeChan⋱ ge	Vs	$ListFieldsBac⋱ kgroundAnd⋱ Perturbed
a$	Ev	SplitGaugeCh⋱ ange	Vspat	$ListFieldsBac⋱ kgroundOnl⋱ y
BackgroundFi⋱ eldMethod	ExtractCompo⋱ nents	SplitMatter	Vv	$ListFieldsPer⋱ turbedOnly
Bs	ExtractOrder	SplitMetric	xPand	$ListOfGauges
Bv	H	SplitPerturbati⋱ ons	k	$ListOfSpace⋱ Types
Conformal	IndicesDown	T	ϵ	$MyRules
ConformalWei⋱ ght	IndicesUp	TensorPropert⋱ ies	ξ	$OpenConsta⋱ ntsOfStructu⋱ re
Connection	Ls	ToBianchiType	ρ	$SortCovDAut⋱ omatic
CS	Lv	ToConformalT⋱ ime	ϕ	$Version
DefConformal⋱ Metric	nt	ToConstantsO⋱ fStructure	φ	$xPertVersion⋱ Expected
DefMatterFields	P	ToCosmicTime	ψ	$xTensorVersi⋱ onExpected
DefMetricFields	RulesSplitCov⋱ DsOfTensor	ToMetric	$CacheRules	
DefProjectedT⋱ ensor	RulesVelocity⋱ Spatial	ToxPand	$Commutecd⋱ Rules	
DefProjectedT⋱ ensorProper⋱ ties	SetSlicing	ToxPandFrom⋱ Rules	$ConformalTi⋱ me	
Disclaimer	SpaceTimesO⋱ fDefinition	V0	$FirstOrderTe⋱ nsorPerturb⋱ ations	