

The radiative transfer for polarized radiation at second order in cosmological perturbations

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This article investigates the full Boltzmann equation up to second order in the cosmological perturbations. Describing the distribution of polarized radiation by using a tensor valued distribution function, the second order Boltzmann equation, including polarization, is derived without relying on the Stokes parameters.

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Introduction

The properties of the cosmic microwave background (CMB) temperature fluctuations depend on both the initial conditions set at the end of the primordial inflationary era, and on their post-inflationary evolution. We describe radiation using the kinetic theory, in which its properties are encoded in a distribution function. The theory of cosmological perturbations around a maximally symmetric space-time enables to solve order by order the evolution equation for the CMB anisotropies. The linear perturbations fail to capture the intrinsic non-linear features of General Relativity which can enter both the initial conditions and the evolution, and we thus need at least the second order if we want to estimate the bispectrum in the CMB and better reveal the physics of the primordial universe. Indeed, the bispectrum can only be generated either from non-Gaussian initial conditions set by inflation or by secondary effects during the subsequent evolution, and though standard scalar field slow-roll inflation predicts negligible amounts of non-Gaussianity [1], the latest observations of the CMB [2, 3] indicate that there might be a non-vanishing bispectrum. We thus need to extend the program followed at first order in perturbations up to second order, that is to build a full set of second order gauge invariant perturbation variables and derive the perturbed Boltzmann and Einstein equations which determine their dynamics. In the fluid limit, the gauge issue was studied in Ref. [4] and gauge invariant variables were built in Refs. [5, 6] up to second order in perturbations. This fluid approximation has already been used to understand the general form of the bispectrum on small scales generated by evolutionary effects in Ref. [7]. As for the kinetic theory, the gauge issue was studied in our previous paper [8], and the evolution equations through free-streaming and Compton collision with free electrons were derived up to second order in Refs. [9, 10, 11, 12], but it is so far restricted to unpolarized radiation, which is inconsistent since Compton scattering does generate polarization. This letter summarizes the full second order derivation of the radiation transfer, including polarization, detailed in Ref. [13].

Kinetic theory with polarization

Describing polarized radiation The momentum of photons is usually decomposed on an orthonormal basis, that is using a tetrad field defined by $\mathbf{e}_a \cdot \mathbf{e}_b \equiv e_a^\mu e_b^\nu g_{\mu\nu} = \eta_{ab}$, $\mathbf{e}^a \cdot \mathbf{e}^b \equiv e_a^\mu e_b^\nu g^{\mu\nu} = \eta^{ab}$. We use Greek indices ($\mu, \nu, \rho \dots$) for abstract indices and the beginning of the Latin alphabet ($a, b, c \dots = 0 \dots 3$) for tetrad labels. We also use $i, j, k \dots = 1 \dots 3$ for the spatial type tetrads and we reserve the label o for the timelike vector. As for the labels associated with the coordinates, we use $A, B, C, \dots = 0 \dots 3$, and $I, J, K, \dots = 1 \dots 3$, with the time component labeled by O .

A momentum can then be written $\mathbf{p} = p^a \mathbf{e}_a = p^o \mathbf{e}_o + p^i \mathbf{e}_i$, and decomposed between the energy p^o and the spacelike direction unit vector \mathbf{n} according to $p^\mu = p^o (e_o^\mu + n^\mu)$ with $n_\mu e_o^\mu = 0$. This decomposition can be used to define the screen projector

$$S_{\mu\nu}(\mathbf{p}) = g_{\mu\nu} + e_o^\mu e_o^\nu - n_\mu n_\nu, \quad (1)$$

which projects on the space both orthogonal to \mathbf{e}_o and to the direction \mathbf{n} . The radiation is represented by a Hermitian tensor valued distribution function also called polarization tensor [14, 15] satisfying $p^\mu F_{\mu\nu}(p^a) = 0$. The screen-projected distribution function, $f_{\mu\nu}(p^a) = S_\mu^\rho S_\nu^\sigma F_{\rho\sigma}(p^a)$, has four degrees of freedom which can be split according to

$$f_{\mu\nu} \equiv \frac{1}{2} I S_{\mu\nu} + P_{\mu\nu} + \frac{i}{2} V \epsilon_{\mu\nu\sigma} n^\sigma, \quad (2)$$

where $\epsilon_{\mu\nu\sigma} \equiv e_o^\rho \epsilon_{\rho\mu\nu\sigma}$, with $\epsilon_{\rho\mu\nu\sigma}$ the space-time fully antisymmetric tensor, and where we have omitted for simplicity of notation the dependence in p^a . $P_{\mu\nu}$, which encodes the degree of linear polarization, is real, symmetric and trace free, as well as orthogonal to \mathbf{e}_o and \mathbf{n} . It has two degrees of freedom, usually described by the Stokes functions Q and U . I and V are respectively the intensity (or distribution function) for both polarizations and the circular polarization. The brightness is then defined as

$$\mathcal{I}(n^i) \equiv \frac{4\pi}{(2\pi)^3} \int I(p^o, n^i) (p^o)^3 dp^o, \quad (3)$$

and we can define similarly \mathcal{V} and $\mathcal{P}_{\mu\nu}$. The remaining dependence in \mathbf{n} , can be further expanded in multipoles using projected symmetric trace-free (PSTF) tensors, where projected means that they are orthogonal to \mathbf{e}_o . For instance, using the notation $n^{\underline{i}\ell} \equiv n^{i_1} \dots n^{i_\ell}$ and $n^i \equiv n^\mu e_\mu^i$, \mathcal{I} can be expanded in

$$\mathcal{I}(n^i) = \sum_{\ell=0}^{\infty} \mathcal{I}_{\underline{i}\ell} n^{\underline{i}\ell}, \quad (4)$$

where the $\mathcal{I}_{\underline{i}\ell} \equiv \mathcal{I}_{i_1 \dots i_\ell}$ are PSTF, and where conventionally, the lowest multipole, i.e. corresponding to $\ell = 0$, is noted \mathcal{I}_0 . A similar expansion can be performed on \mathcal{V} . As for $\mathcal{P}_{ab} \equiv \mathcal{P}_{\mu\nu} e_a^\mu e_b^\nu$, its non vanishing components can be expanded in electric and magnetic type multipoles according to [14, 15]

$$\mathcal{P}_{ij}(n^a) = \sum_{\ell=2}^{\infty} \left[\mathcal{E}_{ijk_{\ell-2}} n^{k_{\ell-2}} - n_m \epsilon^{ml} \mathcal{B}_{(ij)lk_{\ell-2}} n^{k_{\ell-2}} \right]^{\text{TT}} \quad (5)$$

where the notation TT denotes the transverse (to \mathbf{n}) symmetric trace-free part. We have used $(..)$ for the symmetrization of indices, and we will also use $[..]$ for the antisymmetrization.

Transformation rules under a change of frame This description depends on the tetrad chosen to define the polarization tensor, the energy and the momentum direction. Under a local boost parameterized by $v^i \equiv v^\mu e_\mu^i$, the tetrad transforms to $\tilde{e}_a = e_b \Lambda^b_a$, where the components of the transformation are given by $\Lambda^o_o = \gamma \equiv (1 - \mathbf{v} \cdot \mathbf{v})^{-\frac{1}{2}}$, $\Lambda^o_i = -\gamma v_i$ and $\Lambda^i_j = \delta^i_j + [\gamma^2 / (\gamma + 1)] v^i v_j$. Thus, the magnitude and the direction unit vector of the photon momentum transform to

$$\begin{aligned} p^{\tilde{o}} &\equiv \mathbf{p} \cdot \tilde{\mathbf{e}}^o = \gamma p^o (1 - \mathbf{n} \cdot \mathbf{v}), \\ \tilde{n}^{\tilde{i}} &\equiv \tilde{\mathbf{n}} \cdot \tilde{\mathbf{e}}^i = \frac{1}{\gamma(1 - \mathbf{n} \cdot \mathbf{v})} \left[n^i + \frac{\gamma^2}{(1 + \gamma)} \mathbf{n} \cdot \mathbf{v} v^i - \gamma v^i \right]. \end{aligned} \quad (6)$$

This implies that

$$\tilde{S}_{\mu\nu} = \tilde{S}_\mu^\rho \tilde{S}_\nu^\sigma S_{\rho\sigma}, \quad \tilde{n}^\mu \tilde{\epsilon}_{\mu\rho\sigma} = n^\mu \epsilon_{\mu\alpha\beta} \tilde{S}_\rho^\alpha \tilde{S}_\sigma^\beta. \quad (7)$$

Since the screen-projected polarization tensor transforms to

$$\tilde{f}_{\mu\nu}(p^{\tilde{o}}, \tilde{n}^{\tilde{a}}) = \tilde{S}_\mu^\alpha \tilde{S}_\nu^\beta f_{\alpha\beta}(p^o, n^a), \quad (8)$$

then it can be checked from the decomposition (2) and the transformation rules (7) that I and V transform as scalars, that is $\tilde{I}(p^{\tilde{o}}, \tilde{n}^{\tilde{a}}) = I(p^o, n^a)$, $\tilde{V}(p^{\tilde{o}}, \tilde{n}^{\tilde{a}}) = V(p^o, n^a)$, and that $P_{\mu\nu}$ transforms as $f_{\mu\nu}$. The differential solid angle of the momentum direction transforms according to $d\tilde{\Omega} = [\gamma(1 - \mathbf{v} \cdot \mathbf{n})]^{-2} d\Omega$, and thus the transformation rules of the energy integrated multipoles of the brightness can be deduced to be given by

$$\tilde{\mathcal{I}}_{\underline{i}\ell} = \frac{1}{\Delta_\ell} \int d\Omega [\gamma(1 - \mathbf{v} \cdot \mathbf{n})]^2 \sum_{\ell'=0}^{\infty} \mathcal{I}_{\underline{j}\ell'} n^{j_{\ell'}} \tilde{n}_{(\underline{i}\ell)}, \quad (9)$$

where $\Delta_\ell \equiv (4\pi\ell!)/[(2\ell + 1)!!]$ and $\langle \dots \rangle$ is the notation for the symmetric trace-free part. The transformation rules of the electric and magnetic multipoles are detailed in Refs. [13, 15].

The Boltzmann equation The evolution of the polarization tensor is dictated by the Boltzmann equation

$$L[f_{ab}] \equiv S_a^c S_b^d \left[p^h \nabla_h f_{cd} + \frac{\partial f_{cd}}{\partial p^h} \frac{dp^h}{ds} \right] = C_{ab}, \quad (10)$$

where $L[]$ is the Liouville operator and C_{ab} the collision tensor. It can be shown that the Liouville operator preserves the decomposition of $f_{\mu\nu}$ in an antisymmetric part (V), a trace (I) and a symmetric traceless part ($P_{\mu\nu}$), that is

$$L[f_{ab}] = \frac{1}{2} L[I] S_{ab} + L[P_{ab}] + \frac{i}{2} L[V] n^c \epsilon_{cab}. \quad (11)$$

Cosmological perturbations

Perturbation of the metric We assume that, at lowest order, the universe is well described by a Friedmann-Lemaître space-time (FL) with Euclidian spatial sections. The most general form of the metric for an almost FL universe is

$$ds^2 = a(\eta)^2 \left[-(1 + 2\Phi)d\eta^2 + 2\omega_I dx^I d\eta + h_{IJ} dx^I dx^J \right],$$

where η is the conformal time for which the corresponding index is O , and $a(\eta)$ is the scale factor. We define the Hubble parameter by $\mathcal{H} \equiv a'/a$ where a prime denotes a derivative w.r.t. η . We work in the Poisson gauge though, as already mentioned, a complete treatment of perturbations can be performed by defining gauge-invariant variables [6, 8], and we perform a scalar-vector-tensor decomposition in $\omega_I = \partial_I B + B_I$, $h_{IJ} = (1 - 2\Psi)\delta_{IJ} + 2H_{IJ}$, where B_I and H_{IJ} are transverse ($\partial^I B_I = \partial^I H_{IJ} = 0$), and H_{IJ} is traceless ($H^I_I = 0$). Each of these variables can be split in first and second-order parts as $W = W^{(1)} + \frac{1}{2}W^{(2)}$, and if a quantity has also a background value it is noted \bar{W} . First-order variables are solutions of first-order equations, whereas second-order equations will involve purely second-order terms, e.g. $W^{(2)}$ and terms quadratic in the first-order variables, e.g. $[W^{(1)}]^2$ for which we will omit the order superscript. We neglect the first order vector modes ($B_I^{(1)} = 0$) which decay and the first order tensor modes ($H_{IJ}^{(1)} = 0$) which are expected to be very small in standard models of inflation.

Perturbations of tetrads and Ricci rotation coefficients The perturbed tetrad can be expressed in function of the background tetrad in the generic form by $\mathbf{e}_a^{(n)} = R_a^{(n)b} \bar{\mathbf{e}}_b$, $\mathbf{e}^{b(n)} = \bar{\mathbf{e}}^a S_a^{(n)b}$. The symmetric part $R_{(ab)}^{(n)}$ is constrained

by the normalization conditions of the tetrad and thus expressed in function of the metric perturbations. We are free to choose the antisymmetric part $R_{[ab]}$, since it corresponds to the Lorentz transformation freedom (boost and rotation). We require $e^o \sim d\eta$ [16], which is equivalent to choose $R_{io}^{(n)} = S_{io}^{(n)} = 0$ for any n . We also fix the rotation by requiring $R_{[ij]}^{(n)} = S_{[ij]}^{(n)} = 0$. This procedure means that our tetrad is adapted to observers whose velocity is always orthogonal to constant time hypersurfaces. We then choose a background tetrad adapted to our coordinates, i.e. $\bar{e}_b^C = \delta_b^C/a$ and $\bar{e}_C^b = a\delta_C^b$, which finally enables us to express the perturbations of the tetrad in function of the metric perturbations. The perturbations of the Ricci rotation coefficients $\omega_{abc} \equiv \eta_{bd}e^d_\nu e_a^\mu \nabla_\mu e_c^\nu$, needed to express the covariant derivative in the tetrad basis, follows directly from the perturbation of the tetrad, and the components used in this paper are $\omega_{oij}^{(1)} = 0$, and

$$\begin{aligned} \omega_{jik}^{(1)} &= -\frac{2}{a} \partial_{[K} \Psi^{(1)} \delta_{I]J} \\ \omega_{ooi}^{(2)} &= \frac{1}{a} \left[-\partial_I \Phi^{(2)} + 2(2\Phi - \Psi) \partial_I \Phi \right], \\ \omega_{ioj}^{(2)} &= \frac{1}{a} \left[\partial_{(I} B_{J)}^{(2)} - H_{IJ}^{(2)'} + (\mathcal{H}\Phi^{(2)} + \Psi^{(2)'}) \delta_{IJ} \right. \\ &\quad \left. - (3\mathcal{H}\Phi + 2\Psi') \Phi \delta_{IJ} + 4\Psi\Psi' \delta_{K(J} \delta_{I)}^K \right]. \end{aligned} \quad (12)$$

The perturbed Liouville operator

In practice, we want to express the Boltzmann equation in function of η , since we want to perform an integration on coordinates. We should thus multiply Eq. (10) by $ds/d\eta = 1/p^O$. However, there is no point multiplying with the full expression of $ds/d\eta$ since it would then bring metric perturbations in the collision tensor. Instead, we multiply only by $(ds/d\eta)^{(0)} = (1/p^O)^{(0)} = a/p^o$, and we will use the notation $L^\#[\] \equiv L[\](ds/d\eta)^{(0)}$ and a similar definition for the collision term. Since we are also exclusively interested in the dynamics of the brightness, we focus our attention on the energy integrated Liouville operator $\mathcal{L}^\#[\]$ defined similarly to Eq. (3), and we use a similar definition for $\mathcal{C}_{ab}^\#$. The perturbation of the Liouville operator up to second order is of the form

$$\begin{aligned} \mathcal{L}^\#[\bar{\mathcal{X}}, \mathcal{X}^{(1)}, \mathcal{X}^{(2)}] &= \bar{\mathcal{L}}^\#[\bar{\mathcal{X}}] + \mathcal{L}^{\#(1)}[\bar{\mathcal{X}}, \mathcal{X}^{(1)}] \\ &\quad + \frac{1}{2} \left\{ \mathcal{L}^{\#(2)}[\bar{\mathcal{X}}, \mathcal{X}^{(2)}] + \mathcal{L}^{\#(1)(1)}[\bar{\mathcal{X}}, \mathcal{X}^{(1)}] \right\}, \end{aligned} \quad (13)$$

where \mathcal{X} stands for either \mathcal{I} , \mathcal{V} or \mathcal{P}_{ab} . A similar decomposition is performed for the collision term. Since $\mathcal{L}^{\#(2)}[\]$ contains the terms linear in purely second order variables, it has the same functional form as $\mathcal{L}^{\#(1)}[\]$, and consequently we only need the expressions of $\mathcal{L}^{\#(2)}[\]$ and $\mathcal{L}^{\#(1)(1)}[\]$ to report the Boltzmann equation up to second order in perturbations.

We first need the perturbation of the trajectory at first order which is $(dx^I/d\eta)^{(1)} = (p^I/p^O)^{(1)} = n^i(\Phi + \Psi)$. Then, from the perturbations of the Ricci rotation coefficients (12), we deduce the perturbations of the geodesic equation which are given by

$$\left(\frac{dp_a}{ds} \right)^{(n)} + \omega_{bac}^{(n)} p^c p^b = 0. \quad (14)$$

This is used to obtain the second order evolution equation for the energy and the first order evolution equation for the direction (the lensing equation) which read

$$\begin{aligned} \left(\frac{dp^o}{d\eta} \right)^{(2)} &= p^o \left[-n^i \partial_I \Phi^{(2)} + \left(\partial_I B_J^{(2)} - H_{IJ}^{(2)'} \right) n^i n^j \right. \\ &\quad \left. + \Psi^{(2)'} + 2(\Phi - \Psi) n^i \partial_I \Phi + 4\Psi\Psi' \right], \end{aligned} \quad (15)$$

$$\left(\frac{dn^i}{d\eta} \right)^{(1)} = -S^{ij} \partial_J (\Psi + \Phi). \quad (16)$$

This is all what is required to obtain from Eq. (10) the second order Liouville operator for $\mathcal{X} = \mathcal{I}, \mathcal{V}, \mathcal{P}_{ab}$

$$\begin{aligned} \mathcal{L}^{\#(2)}[\bar{\mathcal{X}}, \mathcal{X}^{(2)}] &= \frac{\partial \mathcal{X}^{(2)}}{\partial \eta} + n^j \partial_J \mathcal{X}^{(2)} + 4\mathcal{H}\mathcal{X}^{(2)} \\ &\quad - 4\delta_{\bar{\mathcal{X}}}^{\mathcal{I}} \bar{\mathcal{X}} \left[-n^j \partial_J \Phi^{(2)} + \Psi^{(2)'} + \left(\partial_I B_J^{(2)} - H_{IJ}^{(2)'} \right) n^i n^j \right], \end{aligned} \quad (17)$$

$$\begin{aligned} \mathcal{L}^{\#(1)(1)}[\bar{\mathcal{X}}, \mathcal{X}^{(1)}] &= 2(\Phi + \Psi) n^i \partial_I \mathcal{X}^{(1)} \\ &\quad + 8\mathcal{X}^{(1)} (n^i \partial_I \Phi - \Psi') - 8\delta_{\bar{\mathcal{X}}}^{\mathcal{I}} \bar{\mathcal{X}} \left[(\Phi - \Psi) n^i \partial_I \Phi + 2\Psi\Psi' \right] \\ &\quad - 2S^{ij} \partial_J (\Psi + \Phi) \frac{\partial \mathcal{X}^{(1)}}{\partial n^i} - 2\Phi \mathcal{L}^{\#(1)}[\bar{\mathcal{X}}, \mathcal{X}^{(1)}], \end{aligned} \quad (18)$$

where it is implied that for $\mathcal{X} = \mathcal{P}_{ab}$, according to Eqs. (10) and (11), this expression also needs to be screen projected. The notation $\delta_{\bar{\mathcal{X}}}^{\mathcal{I}}$ has also been introduced in terms involving $\bar{\mathcal{X}}$ to remind that, due to the symmetries of the background space-time, there is no circular or linear polarization at the background level.

The collision term

Electrons without bulk velocity We consider the case where the free electrons have only a bulk velocity and no thermal dispersion in their velocity distribution. Since all electrons have the same velocity, we choose to align the first vector of the tetrad \mathbf{e}_o with this bulk velocity, that is to work in the baryons rest frame. In that case, we can neglect the terms arising from the recoil of electrons, since they will be of order $T_r/m_e \sim 10^{-6}$ around recombination, where T_r is the temperature of radiation, and consequently they do not contribute to the bispectrum. The Thomson approximation is thus sufficient and, in this case, the collision tensor is known and reads [15, 17]

$$C_{ij}^\#(p^a) = \tau' \left[\frac{3}{2} \int \frac{d\Omega'}{4\pi} S_i^k S_j^l f_{kl}(p'^a) - f_{ij}(p^a) \right], \quad (19)$$

where $\tau' = an_e\sigma_T$, with n_e the electrons number density, and σ_T the Thomson cross section. Explicitly, for the energy integrated collision term we obtain

$$\begin{aligned} \frac{C_{ij}^\#(n^q)}{\tau'} &= \left[-\mathcal{P}_{ij}(n^q) - \frac{1}{10}\mathcal{I}_{ij} + \frac{3}{5}\mathcal{E}_{ij} \right]^{\text{TT}} \\ &+ \frac{1}{2}S_{ij} \left[-\mathcal{I}(n^q) + \mathcal{I}_\emptyset + \frac{1}{10}\mathcal{I}_{kl}n^kn^l - \frac{3}{5}\mathcal{E}_{kl}n^kn^l \right] \\ &+ \frac{1}{2}i\epsilon_{ijk}n^k \left[-\mathcal{V}(n^q) + \frac{1}{2}\mathcal{V}_ln^l \right]. \end{aligned} \quad (20)$$

We clearly see on this expression that the circular polarization is not excited and thus remains null if so initially. This explains why we did not bother reporting the corresponding part of the Liouville operator, and from now on we neglect it. Now, if the electrons have a thermal velocity, which is in fact the case, the previous expressions has corrections of order $T_e/m_e \simeq T_r/m_e$ where T_e is the temperature of electrons, but as for the recoil term, we can discard them as they will not contribute to the bispectrum.

Electrons with bulk velocity If the distribution of electrons has a bulk velocity relative to the cosmological frame, then we just need to transform the result obtained in the baryons rest frame to the cosmological frame, using the transformation rules (6) (8) and (9) and remembering that the energy integration is done according to Eq. (3). Keeping only the terms which can contribute eventually to the second order expansion, we obtain

$$\begin{aligned} \frac{C_{ij}^\#(n^q)}{\tau'} &= \frac{1}{2}S_{ij} \left[-\frac{1}{2}\mathcal{I}_kn^kn\cdot\mathbf{v} + \frac{1}{2}\mathcal{I}_{kl}n^kn^ln\cdot\mathbf{v} \right. \\ &+ \left(-\mathcal{I}(n^q) + \mathcal{I}_\emptyset + \frac{1}{10}\mathcal{I}_{kl}n^kn^l \right) (1 - \mathbf{n}\cdot\mathbf{v}) - \frac{1}{2}\mathcal{I}_kv^k \\ &+ \frac{1}{5}(6\mathcal{E}_{kl} - \mathcal{I}_{kl})n^kv^l + 4\mathcal{I}_\emptyset\mathbf{n}\cdot\mathbf{v} - \mathcal{I}_\emptyset\mathbf{v}\cdot\mathbf{v} + 7\mathcal{I}_\emptyset(\mathbf{v}\cdot\mathbf{n})^2 \\ &\left. - \frac{3}{5}\mathcal{E}_{kl}n^kn^l(1 + 5\mathbf{v}\cdot\mathbf{n}) - \frac{6}{5}\mathcal{B}_{kl}\epsilon^{mpl}v_p n_m n^k \right] \\ &+ \left[\mathcal{P}_{ij}(n^q)(\mathbf{n}\cdot\mathbf{v} - 1) + \frac{6}{5}\mathcal{B}_i^l\epsilon_{jpl}v^p + \frac{\mathcal{I}_iv_j}{2} - \mathcal{I}_\emptyset v_i v_j \right. \\ &\left. + (6\mathcal{E}_{ik} - \mathcal{I}_{ik})\frac{n^k v_j}{5} + \left(\frac{3}{5}\mathcal{E}_{ij} - \frac{\mathcal{I}_{ij}}{10} \right) (1 + 3\mathbf{v}\cdot\mathbf{n}) \right]^{\text{TT}}. \end{aligned} \quad (21)$$

This expression agrees with the results obtained for the unpolarized case [9, 10, 11, 12] (see details in Ref. [13]). The perturbative expansion of the form (13) can then be easily read. The expression of the collision tensor together with the Liouville operator obtained in Eqs. (17) and (18) are our key results. It is then possible to shift to Fourier space and expand the Liouville operator and the collision tensor either in PSTF multipoles, or in normal modes components [18] which are better suited for

numerical integration. Details of these extractions can be found in Ref. [13]. Hence it offers, in principle, the possibility to integrate numerically the Boltzmann hierarchy up to second order, though it is expected to be very time consuming, and approximate schemes should be developed in order to focus on the dominant effects. Computing the effects of non-linear evolution has now become extremely important since it has been recently claimed that a non-zero non-Gaussianity in the CMB had been detected [19]. Since the precision of the CMB measurements is going to increase significantly in the forthcoming missions, disentangling the primordial non-Gaussianity, which delivers valuable information on the primordial universe, from the subsequent non-linear effects is a necessary task in order to constrain better from inflation our theories of high energy physics.

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- [1] J. Maldacena, JHEP **0305**, 013 (2003).
- [2] E. Komatsu, J. Dunkley, M. R.olta, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. Limon, L. Page, et al. (2008), 0803.0547.
- [3] K. M. Smith, L. Senatore, and M. Zaldarriaga (2009), 0901.2572.
- [4] M. Bruni, S. Matarrese, S. Mollerach, and S. Sonego, Class. Quant. Grav. **14**, 2585 (1997).
- [5] K. A. Malik and D. Wands, Class. Quant. Grav. **21**, L65 (2004).
- [6] K. Nakamura, Prog. Theor. Phys. **117**, 17 (2007).
- [7] C. Pitrou, J.-P. Uzan, and F. Bernardeau, Phys. Rev. D **78**, 063526 (2008).
- [8] C. Pitrou, Class. Quant. Grav. **24**, 6127 (2007).
- [9] S. Dodelson and J. M. Jubas, Astrophys. J. **439**, 503 (1995).
- [10] W. Hu, D. Scott, and J. Silk, Phys. Rev. D **49**, 648 (1994).
- [11] N. Bartolo, S. Matarrese, and A. Riotto, JCAP **0606**, 024 (2006).
- [12] R. Maartens, T. Gebbie, and G. Ellis, Phys. Rev. D **59**, 083506 (1999).
- [13] C. Pitrou, submitted to Class. Quant. Grav. (2008), 0809.3036.
- [14] A. Challinor, Phys. Rev. D **62**, 043004 (2000), astro-ph/9911481.
- [15] C. G. Tsagas, A. Challinor, and R. Maartens (2007), 0705.4397.
- [16] R. Durrer, Fund. Cosmic. Phys. **15**, 209 (1994).
- [17] J. Portsmouth and E. Bertschinger (2004), astro-ph/0412094.
- [18] W. Hu and M. White, Phys. Rev. D **56**, 596 (1997).
- [19] A. P. S. Yadav and B. D. Wandelt, Phys. Rev. Lett. **100**, 181301 (2008).