# Graph representation of balance sheets: from exogenous to endogenous money 

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#### Abstract

The nature of monetary arrangements is often discussed without any reference to its detailed construction. We present a graph representation that allows for a clear understanding of modern monetary systems. First, we show that systems based on commodity money are incompatible with credit. We then study the current chartalist systems based on pure fiat money, and we discuss the consolidation of the central bank with the Treasury. We obtain a visual explanation about how commercial banks are responsible for endogenous money creation whereas the Treasury and the central bank are in charge of the total amount of net money. Finally we draw an analogy between systems based on gold convertibility and currency pegs to show that fixed exchange rates can never be maintained.


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## Introduction

Understanding the fundamental nature of money is paramount for macroeconomic models. Indeed, over the past century the description of money was central to the building of simplified macroeconomic models, and monetary economics became a field of macroeconomics in itself. The classicals first postulated that money was neutral and avoided the need for any suitable description. The new classicals, and most prominently the monetarists reached a more nuanced version with the quantity theory of money, stating that money was neutral in the long run. Again, the debate was not on the structure of the monetary system but rather on its implications for macroeconomic theories. The debate between neoclassicals and Keynesians led instead to theories of money being addressed as theories of interest rates.

There is certainly no unique theory of money, since money is not a universal concept. Indeed, any monetary arrangement in a society is different and leads to a specific description. We should describe the various possibilities of monetary systems rather than talk about a unique theory of money, including those which have been or are realized, but also allow for the description of possible systems not yet realized in practice, as required by the general democratic debate. This is nicely summarized by Minsky (1996, p1): "[...] relevant theory is not a compendium of propositions derived from axioms assumed to be universally true: economic theory is not a subdivision of mathematics. Relevant theory is the result of the exercise of imagination and logical powers on observations that are due to experience: it yields propositions about the operation of an actual economy".

Monetary systems can be divided in two major categories: metallism and chartalism. Oversimplifying, one can say that metallism is the description of monetary systems in which money

[^0]is backed by a real asset, often gold, whereas chartalism is the description of monetary systems based on pure fiat money that originates from the state's ability to raise taxes (see Bell (2001) for a historical perspective and Knapp (1924); Lerner (1947); Mitchell-Innes (1913) for seminal papers). Since the end of the Bretton-Woods system in 1971 , which was already a remote form of convertibility, this latter descriptive framework has received renewed attention and is often named neo-chartalism in that respect (Wray, 1990).

In this article, we emphasize that all monetary arrangements are intrinsically weighted graphs, because accounting is performed with double entry balance sheets. As Minsky explained Minsky, 1992, p12): "A capitalist economy can be described by a set of interrelated balance sheets and income statements", implying that graphs are the natural theoretical tool to describe monetary systems. Graphs are better suited for representing the global structure of financial systems than lists of individual balance sheets, since they allow for a clear visualisation of the relations between members of the system. Such graphs are rather common in representations of flows in macroeconomics, but are generally absent from the description of stocks, although they have already been extensively used to study the systemic risks in networks of commercial banks [see e.g. Gai \& Kapadia (2010); Sheldon \& Maurer (1998); Upper (2001); Upper \& Worms (2004); Wells (2004)]. In this article, we argue that graphs are the natural language in which discussions about the nature of monetary systems should be spoken. By representing a complex structure in a very compact form, graphs depict information clearly, just like a very deep equation carries a powerful meaning in just one line, and this allows for clearer representations of the global architecture of monetary systems. Weighted graphs open the door to discussion of major topics associated with monetary systems, as they allow for an unambiguous and visual representation of the monetary arrangements.

In a physical system with a high number of particles, there are complex structures that emerge from simple local laws, and they are quite often very difficult to understand. Similarly, the financial relations between all actors of a monetary system are complex and simple at the same time. They are simple because double-entry bookkeeping is understood at all levels of the system, but there are complex financial structures which emerge out of the simple local laws of accounting, due to a high number of financial interactions. As a result, if the inputs of a graph are understood by all, the different schools of economic thought should agree on the graph representation of any monetary structure. Indeed, we show that each given financial situation can be summarized by drawing the corresponding graph, and this is crucial to discuss controversial topics. In particular, we illustrate $i$ ) how public deficits is the source of money creation, ii) that gold convertibility and currency boards are inherently incompatible with credit, and iii) that credits do not originate from deposits, all these subjects allowing to discuss further the nature of money. Disagreements therefore lie in the interpretation, that is on the words we drape around a given graph. We will argue that these differences of interpretation are related to the various possible consolidations which can be made in a monetary system, when trying to grasp the complex emerging structures.

## I. GENERAL FORMALISM FOR GRAPHS OF MONETARY ARRANGEMENTS

## A. Commodities and units of account

A general monetary arrangement is a collection of debts between various members of the system. However, before describing how this system of debts is organized, one must first specify what is owed in these debts, and we thus need an appropriate description for ownership. The simplest item that can be possessed and owed is a commodity and in that case, the unit of account (UoA) is any natural unit for that specific commodity, e.g. a unit of mass for gold.

The simplest description consists of representing the owners together with the amounts they
own. However, in order to unify the representation of ownership with the representation of debts that we will include later, it proves useful to depict the representation of ownership on a graph. A graph can be thought intuitively as a set of points, the vertices, some of which are linked by lines, the edges. The totality of a given commodity is represented by one vertex and so are the various owners, the relations of ownership being represented by an edge from the commodity vertex toward the corresponding owner. Both representations are depicted in the left plot of Fig. 1 . In the graph representation, the edges are weighted by the corresponding amounts owned. The resulting graph structure is thus a weighted and oriented graph with a simple tree structure. The oriented edges can be viewed as arrows, and when departing from a vertex they represent a liability, while when incoming on a vertex they represent an asset.

A slightly more complicated structure arises if the entirety of the commodity is completely stored in a bank vault. There is one direct relation of ownership between the commodity and the bank, with the bank issuing corresponding liabilities to its customers. The members of the system either a) directly possess the UoA, or b) own it indirectly through a bank, having a (positively credited) account in that bank. In scenario b, we say that the bank issues an I Owe You (IOU), as a record of the gold placed into the vault. In scenario a, the customers need to physically exchange the commodity. Scenario b is expanded when these indirect ownerships are made through several banks, as in free banking systems. We illustrate this in Fig. 1 .


FIG. 1 Left top: direct commodity ownership with a graph representation. Left bottom: standard representation of direct commodity ownerships without a graph. Middle: indirect ownership of a commodity through a single bank. Right: indirect ownership through several banks, with each bank owning directly the commodity.

## B. Emergence of central banks

Several configurations are bound to appear once customers of different banks start to interact in the economic world and need to exchange to settle payments. In a system of independent banking the banks can ignore each other. This would force every customer to have an account in each bank. Indeed, once a person accepts a payment from somebody having their assets in a given bank, this person would be forced to open a bank account in the same bank to be able to transfer a liability. With decentralized banking, each customer would still have only one bank account in a given bank, but the banks themselves would possess bank accounts with other banks, so there would be oriented edges between the banks. In this system, a bank $A$ owes gold to its customer, either because it has gold in its vault, but also possibly because another bank $B$ owes gold to bank A. This situation can become rather complex, with a maximum of $N(N-1) / 2$ claim-debt relations
between $N$ banks. In fact, a general theorem that whenever there are strictly more that $N-1$ edges between $N$ connected actors, there must be loops, that is paths along the edges of a graph which start and end at the same vertex. It can thus become optimal to simplify the networks of debts and claims between banks, by removing all unnecessary loops. The simplest solution is a system based on centralized banking, where we use a tree graph (a graph with no loop) of unit depth, with one bank at the root acting as a central bank. It is obvious that in that case there would be only $N-1$ edges in this network of $N$ banks. As explained in the next section, any new debt added between two actors in a tree creates a new loop that can be removed to preserve an optimal tree structure. In a last round of cooperation, the banks can decide to send their gold in the vault of the central bank, and keep a claim on it, so as to optimize the cost of gold keeping.

Note that in the case of independent banking and decentralized banking, there is actually a central structure hidden by the possession of gold. In decentralized banking, if all banks settle their debts, they are back to independent banking, and they are united through the claims they have on the commodity. So even if there is no central bank, or no central system, the fact that only one commodity is used is functionally similar to a system with a central bank. Gold acts as the root of a tree graph. The centralized banking system serves to avoid physically displacing heaps of gold by storing it all at the central bank. The use of gold, with its physical exchange, is rooted in the lack of a central organization, as it effectively replaces it.

## C. Payments in a tree graph

If the tree-structure has been clearly identified with a central bank and all other banks directly related to it, then payments between customers of these banks are easily depicted. For each pending payment there is a unique loop created because the payment at this stage is the addition of a new debt. Recall that a tree with $N$ vertices, has $N-1$ edges, so when one edge is added, there is necessarily one loop. In order to maintain the tree structure and to avoid direct claims between customers or between banks, this loop needs to be removed. For that purpose, when a customer $a$ pays a customer $b$, it needs to provide its bank $A$ with the name of the bank $B$ of customer $b$ so that the loop can be identified. Then, we simply need to subtract the amount of the payment in all edges of the loop as illustrated in Fig. 2. In the first panel a pending payment results in a new short-term debt which creates a new loop. Then this loop is emphasized and in the last panel the tree structure is recovered after the loop is removed, and the new balances of debts-claims have been adjusted. Note that we are actually stacking two types of graphs. The red graph represents commodity ownership relations, and the black graph is made of IOUs for that commodity. Note also that commercial banks can decide to keep direct claims as in decentralized banking. In that case, one would have a loop displacement rather than a loop removal, as the direct claim between customers would be transformed into a direct claim between commercial banks. It amounts to a partial settlement since it maintains the tree structure below each bank but not among banks. Note also that with this structure there are several ways to remove the extra loop introduced by the payment of one customer to another one. In the remainder of this article we do not investigate in more details the interbank structure and for simplicity we assume that payments are fully settled. At any time step, there will be payments, and by successively incorporating them, we effectively build the graph of stocks as an addition of payments from an initial condition. Graphs of stock are the position of the monetary system, and graphs of flows are their variation.


FIG. 2 Description of payments via graphs. $C$ stands for the central bank, $B_{1}$ and $B_{2}$ stand for the commercial banks and $1,2,3,4$ are the various customers of the system. All debts/claim relations are reported as weights on the edges of the graphs, and are expressed in the UoA of the underlying commodity fully stored in the central bank.

## D. Representing debts

Debts from customers of the system toward a bank can be incorporated in this framework, as it is a liability for the member and an asset for the bank. They are all based on the same principle: to lend an amount and to ask for a higher repayment at maturity. The difference between the initial amount lent $S$, and the repaid amount can be converted into an interest rate $i$ for a given unit of time, using typical compound interest for the duration of the maturity $D$. Any complicated loan structure can always be considered as a set of independent loans following this same simple structure. In the graph representation, we indicate a debt by $(S, i, D)$ where $S$ is the current amount, $i$ the interest rate and $D$ its maturity. After every unit of time, $(S, i, D)$ is replaced by $(S \times(1+i), i, D-1)$, and when the maturity becomes null, it is transformed into an immediate payment of amount $S \times(1+i)^{D}$ whose loop needs to be removed exactly as described in $\S$ I.C. And just like for a payment, it comes as a reduction of the liability from the bank toward the member that the latter must have earned through previous payments made to him. Because the nature of each debt is different due to variations in interest rates and maturities, this means that we are now considering several types of assets and liabilities within the same graph. By representing several types of debts on the same financial graph, we are actually stacking several weighted graphs on the same set of vertices. The representation of debts is used in $\S$ II.B where we detail credit creation.

## E. Money and net money

The IOUs of the bank are merely a promise to convert them into IOUs of the central bank (and if there is convertibility, these IUO can be converted into gold). These financial assets are equivalent to interest-less debts. By introducing the possibility to have debts, we see that we will have a mixed system with two categories of IOUs. In the first category, we have the IOUs of the central bank, and the IOUs of commercial banks which are promises to deliver IOUs of the central banks, and in the second category we have the IOUs of customers who have borrowed and who promise to repay their debt in the future using IOUs of the first category. Throughout this paper we will make a distinction between money and net money when referring to these IOUs:

- money held by a customer of the system consists in the financial assets, and it is the reflection of IOUs of commercial banks and of the central bank toward the customer;
- net money held by a customer consists in money from which we subtract any liabilities, hence it consists of financial assets minus the customer's financial liabilities, or net assets.

These definitions are expanded to a group of members of the system by direct summation. As we shall detail in $\S$ II.B, any new loan modifies the total money, but conserves the total net money. Therefore attention must be paid to using this precise language. This distinction between money and net money is also related to the various definitions of monetary aggregates that we discuss in § IV.

## F. Consolidations and sub-graphs

Let us draw a closed line in the monetary graph to define a sub-region. We can replace the subregion by a single vertex whose assets and liabilities are depicted by the incoming and outgoing lines of the region. In practice, this means that if we do not need to describe with precision the internal financial structure inside the region, we can ignore it and simplify the graph by using a simple vertex. In accounting, this is called a consolidation and helps in reading the structure of graphs. One can for instance decide to consolidate the financial sector, and thus to reduce all banks to just one bank like in Fig. 3. Through consolidation, all financial relations which start and end inside the consolidated region do not matter for the exterior, so when the banking sector is considered from a consolidated point of view, interbank relation do not matter at all.


FIG. 3 Left: the closed curve identifies the commercial banking sector. Right: consolidation of the commercial banking sector.

Alternatively, instead of erasing all the details of the sub-region, we can focus on it by eliminating the details of the outside region. This amounts to considering a sub-graph, with externals sources and sinks. For instance, in the left plot of Fig. 3, we could focus on the banking sector which sits inside the closed curve. This is exactly what is done by Gai \& Kapadia (2010); Sheldon \& Maurer (1998); Upper (2001); Upper \& Worms (2004); Wells (2004) where the emphasis is on the stability of the banking sector itself.

## G. Relating balance sheets and financial graphs

We have provided the basic tools to represent a financial system with graphs. Since a financial system can also be represented using balance sheets, it is worth discussing why these two approaches are related and why graphs are more convenient.

- Equivalence with balance sheets. A weighted graph is equivalent to an antisymmetric matrix $W$, whose entries correspond to the weights of the graph. For instance, if we have $W_{i j}=$ $-W_{j i}>0$, then this value corresponds to the weight of the directed edge from the vertex $i$ toward the vertex $j$. Balance sheet representations are then just a special way to visualize such antisymmetric matrices. They amount to detailing the matrix column by column, separating the positive and negative entries. For a chosen vertex $j$, the entries of the column are the $W_{i j}$ where $i$ runs over all vertices. If $W_{i j}>0$, then the financial relation is an asset as it corresponds to an IOU from vertex $i$ to vertex $j$, and if $W_{i j}<0$ it is a liability since it corresponds to an IOU from vertex $j$ to vertex $i$. Said differently, for a given vertex the outgoing edges (resp. the ingoing edges) are the liabilities of that vertex balance sheet (resp. the assets of that vertex balance sheet). For each graph representation, the balance sheet picture can be inferred by the examination of the edges attached to each vertex. Conversely the graph and its corresponding matrix can be reconstructed from the list of all balance sheets. We must note that if we have several different types of financial relationships, then we have several types of antisymmetric matrices and thus several types of weighted graphs and several types of entries in the balance sheets. For instance in Fig. 2, we have one graph for the ownership of gold that is partially represented, and one graph for IOUs on gold. In the same way that matrices can be added, graphs can be stacked, and balance sheets can be combined so as to lead to a global representation of the financial relations.
- Advantages of graphs. It is simpler to visualize the global properties of a financial system from a graph representation than with just a list of balance sheets. Indeed with balance sheets the related entries $W_{i j}$ and $W_{j i}=-W_{i j}$ appear twice, since they appear in the balance sheet of vertices $i$ and $j$. For instance if $W_{i j}>0$ it appears in the liabilities for vertex $i$ and in the assets for vertex $j$. The reader must mentally reconnect all these pairs to obtain a clear understanding of all financial relationships. With a graph, the entries $W_{i j}$ and $W_{j i}$ appear naturally as a single weighted edge connecting vertices $i$ and $j$. Financial relationships are thus directly observable without any effort and consolidations are immediate as described in $\S$ I.F. The balance sheets representation is the collection of the individual point of views on a monetary arrangement, whereas the graph representation is the global point of view which allows to grasp the emerging features of the system. If a balance sheet is enough for accountants needing only their local situation with respect to the rest of the monetary arrangement, the graph is necessary to visualize and thus understand the global organization.


## II. PRIVATE BANKING AND ENDOGENOUS MONEY CREATION

Having all the tools to graphically represent financial stocks, we are now ready to investigate the general organisation of monetary systems. In this part, we focus on the specificities of the banking sector and postpone the role of the Treasury to $\S$ III.

## A. Net worth of banks

In this section, we define the net worth of banks so as to be able to investigate in detail the mechanism of money creation through credit creation in the next section. An immediate issue when allowing for debts, is the possibility that the borrower might not be able to repay the loan when it is due. Indeed, the borrower thinks that he will be able to repay in the future because he is going to exchange his work against money, which in turn he will use to reimburse. Once the debt is turned into an immediate payment at maturity as explained in $\S$ I.D, the borrower must have sufficient assets in order to be able to repay the loan.

From a graphical point of view this would be equivalent to a unilateral removal of the debt from a customer to its bank. However, if the bank had initially as many liabilities as assets, this is no more the case after the debt removal, so in general one cannot assume that liabilities and assets are balanced. We thus introduce the concept of net worth of a vertex, that is a bank, which is the difference between assets and liabilities of that vertex.

So far the bank was just a node or a vertex in the graph structure, but behind a bank there is a banker ${ }^{1}$. If a borrower defaults, the banker should pay the corresponding debt for him and this takes the form of a corresponding decreasing of the bank net worth. Conversely, when the borrower reimburses but also pays interest, it does so to the banker and this means that the bank net worth is continuously increased by the interest, and this is also another reason why the introduction of a net worth is necessary. To summarize, the risk of defaulting together with the benefit of charging interest, are affecting in opposite directions the bank net worth.

The notation of consolidation needs to be extended to take into account the net worth of banks or companies. If we draw a closed region that we want to consolidate, we just need to add all the values of the net worths which are inside, and they will add up to the equivalent net worth.

## B. Loans, deposits and banking

In this section, we detail the process of credit creation, and illustrate the well known fact that it is at the origin of money, as bank loans create deposits (Tobin, 1963). This is illustrated in the left plot of Fig. 4. In the first graph of this plot, the customer initially takes out a loan for an amount $S$, with interest rate $i$, and maturity $D$ which is represented on the edge going from customer 2 to bank $B$. As a result, customer 2's liability to bank $B$ has increased from $M_{2}$ to $M_{2}+S$. The bank $B$ also has liabilities toward customer 1 for an amount $M_{1}$. This is reflected in the liabilities from the central bank $C$ to the bank $B$ which sum up to $M_{1}+M_{2}$. The payment that customer 2 intends to make to customer 1 is superimposed as a dashed arrow. In the second graph, after settling the payment, customer 2 is left with an extra liability and customer 1's assets have increased. We observe that the sum of all customers' assets and liabilities owed to the bank is unchanged. Indeed, if we were to consolidate the customers into only one customer, then the total net assets, that is the total debt of the banking sector toward the consolidated customer is unchanged. But since we have decided to call money the financial assets of customers, and not the net assets, we can say that there has been money creation as a result of the act of borrowing and spending.

The reverse process occurs when the loan is to be repaid after $D$ units of time have elapsed. This is illustrated in the right plot of Fig. 4 where we assume for simplicity that the customer has no money $\left(M_{2}=0\right)$. In the first graph the borrower needs to work so as to receive a sufficient payment

[^1]which is settled in the second graph. Since the borrower owes the principal plus interest, the net worth of the banker has increased by an amount $I$ equal to the expected interest. In the third graph, the borrower repays the loan and credit money is destroyed. The net worth of the bank has been increased by the interest $I$ which has been subtracted from the total of the customers' assets. The liability from the central bank to the commercial bank is unchanged. Eventually the bank can redistribute totally or partially its increased net worth among its employees and shareholders in the form of salaries and dividends. This dynamic creation and destruction of money is called the endogenous nature of money.

It is often not intuitive to understand that banks can create money, since one would expect that they would create arbitrary amounts of money for themselves. However what banks can create endogenously is money, not net money. That is if they create money and give it to their bankers and shareholder it comes as a decrease of their net worth, as is evident if they then spend it to pay customers of other banks. It is equivalent to an advance on future salaries and dividends, and banks can only do so with the restriction that the net worth remains positive.


FIG. 4 Left: Creation of money after a loan is granted and spent. Right: Loan repayment and destruction of money.

## C. Interbank payments and financial institutions

In the previous section, we considered customers who could pay each other easily because they have accounts at the same bank. If the borrower spends by paying someone with an account at another bank, this creates a loop in the banking tree structure, which is removed when the payment is actually settled. This is straightforward if the borrower's bank has enough central money as illustrated in the left plot of Fig. 5. In the first graph the bank grants the loan and creates the corresponding deposit. In the second graph we illustrate the result of the payment to a customer in a different bank. This way of lending is typical of financial institutions since in order to have sufficient central money they must have received earlier deposits from customers.

However if the borrower's bank has no central bank money because it never received deposits from customers, this proceeds differently. In the right plot of Fig. 5 we assume that the bank making the loan has no central bank money when it grants the loan in the left graph, but it makes sure to ask for an interest rate higher than the discount rate. This leads to two possibilities once the payment to a customer of a different bank is settled. The first possibility is the borrower's bank is indebted to the central bank, which is illustrated in the middle graph. The central bank will charge interest at the discount rate for this and acts with banks as a bank acts with its customers. The second possibility is that since the second bank now has an excess of central bank money bearing
only the deposit rate, it might prefer to lend it directly to the indebted bank at a higher rate but that is still less than the discount rate, such that both banks benefit from this arrangement. We have not written the net worths of bankers for simplicity, assuming they vanish, but they will positively evolve due to an accumulation of interest payments.

Importantly, since the central bank guarantees the payment of interest on the deposits (possibly zero) of commercial banks at the central bank, but also guarantees that the commercial banks can borrow from the central bank at the discount rate which is of course higher than the rate paid on deposits, then no commercial bank would loan its excess of central bank money at a rate lower than the deposit rate, and would borrow at a rate larger than the discount rate. The central bank thus defines a corridor of rates in which the commercial banks negotiate interbank loans. The interbank rate evolves on a daily basis and the central bank tries to influence it with its monetary policy (see § III.B).

If the bank of the borrower had enough central bank money (due to previously existing depositors) to settle the interbank payment, it acts like a financial institution, as we could think of it as lending the previous deposits of customers. But if the depositing customers decide to withdraw their assets before the loans made are repaid, then the balance sheet of this institution looks again like a usual bank. The difference between a financial institution and a bank is (Tobin, 1963, p10) of degree not of a kind. Finally, we note that the power lies in the institution that allows to be indebted. When a bank goes into debt, the central bank sets the conditions of the loan. In a tree structure, we have a hierarchy with the upper vertices holding a form of power on those situated below.


FIG. 5 A loan is granted and spent to pay a customer of a different bank. In the left plot, the bank which granted the loan has enough central bank reserves, and in the right plot it does not have central bank reserves at all, so it must either borrow to the central bank or in the interbank market.

## D. Bank runs and the nature of money

Let us show why once credit has been included in a monetary system, it becomes incompatible with gold convertibility. In this scenario, the central bank IOUs are the promise to deliver gold. In a monetary arrangement, the sum of net worths of all vertices, that is banks, and leaves, that is customers, reflects all the IOUs issued by the central bank. This is because if we consolidate all banks and customers into a single member, then its assets can only emanate from the central bank given that the consolidation erases the internal structure. If there is no debt at all, and customers have only assets, these assets are exactly reflected by the IOUs of the central bank and everybody
can ask at the same time to convert their wealth into gold. The entire tree would vanish, and we would be back to a situation where everybody has physical shares of the commodity.

However, if we allow for debt among the customers then we must be careful with the fact that money (positive assets) is not net money. If we allow all positive assets to be converted to gold, then we have more claims on gold than gold itself, and the system fails. How is failure possible if we have been so careful to ensure double entry bookkeeping at every stage of our monetary system? It is extremely simple and reflects the true nature of money with respect to net money. Money is now both claims on the gold of the central bank, and claims on the debts of the borrower. It has shifted from its commodity nature to its credit nature. The difference lies in the fact that the debts of borrower are not available immediately since borrowers have to work to be able to repay them. This is illustrated in Fig. 6. In the left plot the central bank holds $M$ units of gold which are reflected into the central bank IOUs toward the commercial banks. The total assets of customers are partially reflected by the gold held at the central bank, and partially by the debts of the borrowers. The orphan edge emerging from the total gold commodity stands for all the gold held outside, e.g. in foreign countries, or held privately, and even the gold which is lost or not yet discovered. If customers ask to convert their assets into gold, they cannot convert all of it. At some point, the central bank needs to stop the convertibility. In the right plot, if the central bank stops gold convertibility, then customers can only convert their assets into central bank IOUs, usually in the form of anonymous paper money.

We interpret this saying that money is partially a right to get gold, and partially a right to ask for the borrowers to work by purchasing their workforce, even though this right does not exist in legal terms. Indeed when a customer with positive assets pays a borrower in exchange of work, the reduction of the loop reduces the assets of the payer, and reduces the liabilities of the borrower. There is net money conservation, but there is money destruction.


FIG. 6 Left: Customers ask to convert their assets into gold until the central bank runs out of gold. Right: if the central bank stops gold convertibility, then customers can only convert their assets (commercial bank IOUs) into anonymous paper money.

It is because of this incompatibility between money and net money, the fact that the first one is only partially a claim on the assets of the central bank, but also partially a claim on the future workforce of borrowers, that convertibility must be abandoned. By allowing customers to go into debt, but by incorporating these debts in the monetary structure, we have shifted the nature of the IOUs in an irreversible manner that calls for the end of convertibility. Conversely, if no bank agreed to generate money endogenously, and if loans where made by customers to customers in the form of bonds that would not be encapsulated into the tree structure, then the IOUs of the banks
would still be claims on the central bank gold reserves. One would have two types of currencies. One would be the IOUs of banks which are eventually redeemable in central bank assets (gold), and then the IOUs of individuals that we could also exchange to settle payments. In some cases we would exchange claims on gold, and in other cases claims on future workforce, but nothing guarantees that they would be traded at par. By incorporating the loans, or at least a major part of the loans in the tree structure, we have obtained a much more liquid system, at the price of abandoning convertibility. As we explain in $\S$ V.B, the same process happens with a currency pegged to an external one, since the external currency acts like gold. Allowing the conversion of money instead of just net money in the external currency, contains the same internal contradiction leading to an unavoidable breakdown of the currency peg. In the remainder of this article, we assume that gold convertibility by the central bank is no more possible, as is the case for the current monetary systems. Finally, note that the hierarchical structure has already been discussed in earlier literature (see e.g. Mehrling (2012)) where it has been understood that it emerges from the need to merge money and credit at all levels so as to obtain liquidity, and we emphasize that this structure is made clear through the use of oriented graphs which are mainly organized in a tree structure.

## E. From commodity to fiat money

The story we presented for the integration of banks in a network is only a pedagogical story, and is by no means history. The presentation adopted here would imply, if taken at face value, that there was first barter, then in a refinement, there was exchange of commodities, and only after came a mild form of fiat money as a claim on commodities (gold) toward banks. This point of view is essentially the metallist point of view or the commodity theory of money. If this might have been true for some societies at some times, there is no proof that this has been a general feature. And as we have shown in $\S$ II.D, it is inconsistent when the banking system allows for the creation of credits.

It is argued by Graeber (2011) that this is rather the exception in history, and that the most generic type of money is credit money which finds its origins in the quantification of moral debts in units of accounts. The gold standard money appears in that perspective as an exception driven by the industrial revolution, and ended in 1971. These general ideas are rooted in the chartalist description of the origin of money, where the UoA for the debts is actually set by the state. It dates back from Mitchell-Innes (Mitchell-Innes, 1913) who emphasized that money is a standard of deferred payment. As the state spends in a chosen and arbitrary UoA, it also sets that this unit should be used for debt repayment, as all individual debts toward society which take the form of taxes have to be redeemed in that state UoA. In that case, this pure fiat money does not derive its value from the market as a commodity, but is rather credit money whose value is initiated by the sovereign states. These ideas have been further developed by Lerner (1947) and Knapp (1924) and are currently revived under the name neo-chartalism (Wray, 2004).

In order to shift from a commodity money to a pure fiat money we just need the central bank to give up on its commitment to convert its IOUs into gold. In that case, any customer can ask for its assets to be converted into central bank's IOUs, usually paper money, but not into gold. The chartalist theory of money argues that everything would behave as usual as long as the state continues to tax in that UoA. It thus argues that taxation is a sufficient condition to impose that the state IOUs are used for debt repayments. The true amount of commodity inside the stateissued coins does not matter, as long as the state has a monopoly on coinage and the ability to tax in this unit. The power of states lies in their ability to tax and neo-chartalists thus refer to such a fiat money system as a sovereign currency.

Forstater (2003) [see also Cottrell et al. (2009)] has recently explored monetization in colonies by colonizing countries, and showed that it was performed exactly following that logic. In many cases, the colonizing countries spent their currency in the colonies, and enforced its use by promising to tax in that same currency. When the US landed in Europe on 6 June 1944, they wanted to impose such a chartalist monetization, and it was strongly opposed by the French government in exile, precisely on the ground that this would amount to colonization (de Gaulle, 1959) and not liberation.

Given that the gold convertibility of the Bretton-Woods agreement has been abandoned in 1971, there is no doubt that the current international system is a chartalist system. In the next part, we detail the role of the state in the monetary arrangements based on such pure fiat money, and we discuss budgetary tools.

## III. STATE REPRESENTATION

## A. Borrowing and spending: Budgetary policy

## 1. Borrowing methods

The state has a bank account at the central bank, called the Treasury, and it feeds it by requiring citizens to pay taxes. Any taxpayer, creates a new loop when he is asked to pay the Treasury, and by the usual removal of the loop described in $\S$ I.C, the Treasury account at the central bank increases, whereas the customer's account and the account of its bank at the central bank decrease accordingly. It is formally equivalent to taxpayers converting their assets into central bank paper money and handing them over to the state. From a chartalist perspective, this taxation drains central bank IOUs out of the private sector, enforcing the need to use and hold them.

By contrast, public spending goes from the Treasury account to individuals. For sure, the state can always tax more than it spends. However, if the state wants to spend more than what it taxes, it needs to create a public debt which can take various forms.

- Money printing: This is the easiest possibility. It consists in increasing unilaterally the amount on the Treasury account. Since there is no more convertibility this is technically possible, because the IOUs of the central bank are not exchangeable with anything.
- Directly borrowing from the central bank: Another possibility would be not to print money, that is to put it on the Treasury account out of thin air, but to lend it to the Treasury. This is exactly as when money is printed, except that now the state also issues a Treasury bond (T-bond) to the central bank. Such a situation can only really work if both the Treasury and the central bank obey the same sovereign power, so that the state actually controls both sides of the deal, and it is only a formal arrangement. If this is the case, then it can borrow as much as it wants. This is fundamentally different from a country borrowing in a foreign currency, and it is the essence of a sovereign currency. And when it needs to reimburse, it can just borrow again what is needed. Eventually, as the bonds remain and are extended or replaced by similar bonds, this is equivalent to money printing. The process of directly borrowing from the central bank is depicted in the left plot of Fig. 7. Initially the net money of customers and bankers is $M$ and the Treasury borrows $S$ to the central bank in the left graph. The right graph depicts the result after spending the amount borrowed, and the total net money of customers and bankers has increased to $M+S$. This increase can be traced to the central bank as we see that the liabilities of the central bank toward the commercial banks now include the spending of the Treasury. Everything happens as if the T-bonds were the new gold from which the central bank originates its liabilities. If we
consider the consolidated state, which would be the Treasury and the central bank merged, there is creation of total net money.


FIG. 7 Left: The Treasury directly borrows from the central bank and spends. Right: The Treasury borrows from the banks, after they have themselves borrowed central bank money with a repurchase agreement, and then spends.

- Borrowing from banks: In this last case, the state is forbidden to borrow directly from the central bank and needs to borrow from commercial banks. The result is that in exchange of a payment to the Treasury, the state will issue a bond, that is an IOU toward those who have paid the Treasury. It can be individuals, but this is unusual. Commercial bankers borrow from the central bank what they want to lend to the Treasury if they do not have enough reserves at the central bank. This latter method is slightly more complicated as banks need to provide a collateral, for instance a previously owned T-bond, in a repurchase agreement and this is illustrated in the right plot of Fig. 7. In the first graph, we assume that the banks already have a T-bond in their possession, and then in the second graph they use it to engage in a repurchase agreement so as to borrow central bank money. At first a commercial bank sells a bond to the central bank against a smaller amount of central bank money, and then it buys it back at its value, the difference between the two amounts being effectively an interest paid. The maturity of the loan is smaller and for this to be interesting, the interest rate needs also to be smaller than the interest served on T-bonds. The collateral is there to make sure the bank will repay the loan in one form or another. Either it repays by buying back the collateral, or it defaults and the central bank keeps it. The result of the repurchase agreement is that instead of directly holding the collateral, the commercial bank holds a claim on it, and we highlighted it by the orange dashed box. In the third graph the commercial bank transfers the amount borrowed previously toward the Treasury in exchange of a newly issued T-bond. In the final graph the Treasury has spent the amount borrowed. Note that the Treasury could have also decided to open an account at the commercial banks so that the amount borrowed would never have been transferred to Treasury, but after spending the result is the same.

When commercial banks lend to the Treasury, the amount of IOUs possessed by the bankers remains constant as can be seen by comparing the first and the third graph of Fig. 7, right plot. However, the nature has shifted from central bank IOUs which promise nothing, to Treasury IOUs which promise central bank IOUs in the future with interests. This new T-bond held in the banks net worth can then in turn be used as a collateral for further borrowing at the central bank, and further lending to the Treasury, without any theoretical
limit. Note also that to obtain a clear visualization, we have consolidated the commercial bank sector and also the customers, but in reality one bank is involved when it lends to the Treasury, and other banks are subsequently involved through interbank payments when the Treasury spends in the various parts of the public sector.

## 2. Net money and financial wealth

As a result of government borrowing and spending, the total net money of customers is increased by the amount spent by the Treasury, $S$ in both cases depicted in Fig. 7. In the case where the Treasury borrows directly from the central bank, this increase of financial wealth is reflected by an increase of liabilities of the central bank toward the commercial banks. Effectively, the central bank converts the IOUs of the Treasury into its own IOUs. However, if the government needs to borrow from commercial banks, the increase of financial wealth has a double origin. Some comes from the liabilities of the central bank, and some comes from the liabilities of Treasury. Effectively, the commercial banks convert the IOUs of the Treasury to their own IOUs which are then redeemable in central bank money. We conclude that when the Treasury borrows directly from the central bank, the T-bonds are converted into net money by the central bank, whereas when the Treasury borrows from a commercial bank, this conversion is made by the commercial banks.

Consolidating the Treasury and the central bank together helps to interpret this. The total net money of customers has increased by the amount spent by the state. In the first case the Treasury IOUs are internal to the consolidated state, whereas by borrowing to banks, the T-bonds now leak out of the consolidated state sector as they are assets of the banks. In both cases the net worth of the consolidated sector of banks and customers has increased, and a natural interpretation is that the net money or the financial wealth of the public sector has increased. However, a different consolidation is possible and one might prefer to consolidate the banks, the customers and the Treasury together in what we consider as the public sector. In that case we observe that the total net worth is unchanged. We realize in this example that the interpretation of a given monetary system crucially depends on the consolidations we perform so as to simplify and describe it. We postpone a detailed discussion about choices of consolidations to $\S$ III.C and III.F.

## B. Open market operations

It can be argued that it would be much more democratic if Treasuries were allowed to borrow directly from their central bank. By electing a government with such a program, we would know what deficit it intends to run and thus how much it will be willing to print, which in the long run is a debate about the possible level of inflation. Instead, it has been argued that decisions made on democratic grounds might be unstable as they are affected by elections. However, the independence of central banks also serves the interest of commercial bankers as we detail now.

In practice, the central bank buys and sells bonds in open market operations ${ }^{2}$. At least it is always doing so with short term T-bonds as part of the conventional monetary policy, and it might decide sometimes to do it as well with longer maturity T-bonds as part of the unconventional monetary policy. This blurs the line between a model where the central bank directly finances the Treasury, and a model where this is done by commercial banks since they result in the same final situation. Indeed, before an open market operation the Treasury owes central bank money to a

[^2]commercial bank, and in the final situation it owes it to the central bank itself, and the central bank money held by the commercial bank has been accordingly increased.

The commercial bank has agreed to dispose of an IOU which bears interest, in exchange for a central bank IOU which bears no interest. However the Treasury will never default on its debt, because the state also runs the central bank which can buy an infinite amount of T-bonds. Said differently, if the interest rates for short term T-bonds start to increase as the commercial banks become more and more reluctant to buy these, the central bank needs to buy as many short term bonds as necessary to ensure the short term interest rates on T-bonds remain at the targeted level. By using these open market operations, a sovereign state running a sovereign currency has the means to ensure that the banks are always willing to buy T-bonds, whatever the deficit is.

However, this system has a drawback. First when the commercial bank bought the T-bond, it asks for interests rates which are at least slightly higher than the interest rate at which they can borrow from the central bank, and make a profit on the difference. As the interest rates departed from the target chosen by the central bank, the latter bought short-term bonds to prevent the short-term rate from increasing. In order to convince a commercial bank to get rid of a financial instrument which is not risky and which bears interest, the only solution is to pay more than the current value of the bond, which amounts to a decrease of the interest rate on those bonds. The bank thus makes an immediate profit instead of a larger profit later. This difference goes directly into the net worth of the banker and amounts to money creation.

To conclude, we reach the same situation where the Treasury had directly sold its bond to the central bank, except that now we have increased the net worth of the bankers. By first selling the bonds to the commercial banks, instead of selling directly to the central bank, the bankers realize a small profit, that is they benefit directly from the operational constraint that the Treasury cannot directly borrow to the central bank.

The commercial banks will always end up with a part of their assets denominated directly in central bank money, which bears no interest, and T-bonds, which bear interest. If we adopt a consolidated state point of view, where we merge the Treasury and the central bank, then the commercial banks have two types of accounts. Deposits which bear no interest, and saving accounts which generate interest, just like standard customers at their bank. In order to control the interest rate, the consolidated state shifts the amounts from the interest-less to the interest-bearing account and vice-versa.

## C. State consolidation

This point of view of consolidating the Treasury and the central bank is usually criticized because the central bank is supposedly fully independent. On the contrary, we can argue that it is also a creature of the state and independence is only superficial since it is decided by the state itself through legal dispositions. Furthermore, it must react automatically with open market operations whenever a new public debt is issued to control interest rates, so that it serves the interest of the Treasury and thus the state.

We must recall that there are two types of short-term interest rates that the central bank can control:

1. the interbank rate (called the Fed funds rate in the US), which is the rate at which the banks can borrow with collaterals at the central bank. This is depicted in the orange dashed box of Fig. 7 (right plot). It is necessarily capped by the discount rate which is the maximum rate at which they can borrow, and above the rate paid on commercial bank deposits, if it exists;
2. the short term bonds rate which is affected by open market operations.

If a bank decides to control the short-term bonds rate, then when conducting outright purchases of short term T-bonds, it increases the central bank liabilities (central bank money), which decreases the interbank rate. So controlling the short-term bonds rate implies to control also the interbank rate.

However, a central bank can decide to control only the interbank rate and not the rate on short-term bonds. For this, it will only conduct repurchase operations, or collateralized loans to commercial banks. This would affect the rate at which commercial banks can borrow, but in general it will not affect directly the rate of the short term T-bonds. Indeed, the Treasury would then borrow like any customer, and banks are free to set the conditions. Nearly all developed countries, except the Eurozone, control both rates and run fully sovereign currencies. However, the European Central Bank (ECB) is in general unable to control interest rates on T-bonds. This is why the ECB performs only repurchase agreements to control the interbank rate, but does not control the short-term rate on bonds via outright purchases of T-bonds, contrary to the Fed monetary policy. As there are several types of bonds, each one of them issued by a different government, the ECB cannot decide which one to buy as it would amount to a form of financial solidarity between the various European states, and this is intentionally avoided in the European Union construction. As the various Treasuries are not helped by the ECB to issue low interest bonds, everything occurs as if they were borrowing in a foreign currency, where the interest rates are set by the bankers, just like for any standard customer. The bankers lending to the Eurozone Treasuries decide what should be the markup rate, that is the difference between the rate at which they borrow, which is effectively the interbank rate set by the central bank, and the rate at which the lend to the various Treasuries. On the contrary, the United Kingdom (UK) or the USA are configured so that their central bank buys whatever is necessary to control the interest rate on bonds, and things are as if the Treasury was borrowing from the central bank, except for the small profit made by bankers due to their intermediation.

To conclude, if the Treasury borrows directly from the central bank, it makes sense to consolidate the Treasury and the central bank. If it borrows from commercial banks, but the central bank controls the short term interest rates on the bonds, the effective theory is nearly the same, and it also makes sense to consolidate the Treasury and the central bank. However, if like in the Eurozone the central bank stops controlling the interest rate on bonds but controls only the interbank rate, the Treasury is treated like a standard customer, as it is treated by the central bank as if it was a foreign Treasury. The consolidation might still be possible formally, since any consolidation can be made as it is just an accounting simplification, but it hides some salient features. Sometimes commercial banks would add only a small markup rate to the T-bonds and one would not see the difference, but in case of crisis the markup rates can start to be huge, on all maturities, even reaching the point where commercial banks stop buying T-bonds as in the recent Eurozone debt crisis.

## D. Monetary policy

Monetary policy tools are very complex and can vary greatly from one country to another. It is by no means the goal of this paper to present them all. The main conventional tools are the control of short term interest rates, and the reserves requirements.

- Conventional monetary policy is about controlling the short term interest rates. However, controlling the interbank rate is not sufficient to control the rate at which customers borrow. Indeed, when commercial banks lend, they need to apply at least the interbank rate as it is
the rate at which they need to borrow when the loan is used to pay outside of the commercial bank, or when a part of the amount loaned is transformed into cash, that is into central bank money. But they also need to apply a markup rate to this basic interbank rate at which they borrow for several reasons. First, the borrowers might default and the bank needs to make sure that it generates enough profits from non-defaulting loans to compensate for the defaulting ones Second commercial banks need to make a profit to cover their running costs as they need to pay at least the salaries of their employees. Finally, since they have borrowed on a short term basis, but they lend on a longer term basis, they need to have a security margin in case the central bank increases the interbank rate. For all these reasons, the effective rate at which the economy is functioning is different from the basic interbank rate chosen by the central bank. If the central bank wants to foster credit with low interest rates, it is as important to set a low interbank rate as to communicate the fact that this interest rate shall remain low, so as to decrease as much as possible the markup rate. Finally, we must stress that the interest rate is not the only criteria to ask for a credit, as decisions are made on much more fundamental economic grounds. Even if the markup rate remains constant, the interest rate set by the central bank is only an indirect tool to control credit and thus the total money.
- Reserve requirements for commercial banks is the second most common monetary tool. In the theory of the money multiplier, the central money held by banks should be a fraction of its liabilities toward its customers, and this fraction should be set by the monetary authorities. It is then assumed that by controlling the amount of central bank money held by the banks, and by fixing the ratio, the central bank could control the total credit, and thus the total money. But this tool cannot be efficient, because the amount of central bank money held by banks is not exogenously set. Indeed what is counted as a reserve for a commercial bank is not its net central money, but its central money. So if the bank does not have enough central money reserves, for instance because it has granted too many loans, it can borrow the reserves needed at most at the discount rate, and more probably at the interbank rate. When the bank does so, at the same time it receives central bank money and increases its reserves to comply with its legal obligation, but it owes it as well at a later date, and this does not count negatively in the reserves. We see that the difference between net central bank money and central bank money is very important. The central bank can control its net liabilities toward the banks, e.g. by performing outright purchases of T-bonds, but it cannot control its liabilities, as these are endogenously determined by the needs of the commercial banks. In practice, banks lend whenever they think it is profitable for them, and if they fail to meet their reserve requirements at the end of the day, they just borrow (directly to the central bank at most at the discount rate or to other banks) what is needed. In a few developed countries (Canada, Australia, UK, Sweden, Norway) there are no more fractional reserves and nothing special happens. As long as the required reserves are not net reserves, they are entirely useless.


## E. Should it stay or Schuld it go: the clash

There is a huge debate on the nature of money and the nature of public debts. Apart from the fact that T-bonds bear interest, they have an intrinsic difference with central bank IOUs, which is that they have a maturity, and are thus bound to disappear through the reimbursement of the Treasury's debt. Let us take a simple case where $20 \%$ of the net deposits is a reflection of the central bank IOUs, and the rest comes from government debt. By construction the customers always have more assets than the debt of the state. So the state could always, at least in theory, run a huge
temporary tax on capitals which would reduce the assets of customers and increase the Treasury account so as to allow debt repayment. This would amount to one year of GDP since nowadays public debts are of that order. The payment of the tax is illustrated in the left plot of Fig. 8, In the first graph we identify the pending tax payment, which is then used to credit the Treasury account in the second graph. In the third graph this is used to reimburse the public debt. As a result the net money would be reduced by that amount, and within our example the financial wealth on deposit and saving accounts would be divided by five. Since 2011, this is essentially what has been implemented in European countries where it has been decided to reduce the public debt at any cost. People might be fine with the reduction of their financial wealth because they believe that in the long run prices should decrease, or even changing their saving habits so that it does not happen. But we think that it is more likely that the private sector will adhere to its preexisting savings habits, meaning that everybody tries to spend less, causing deflation and possibly after recession. So by ignoring this, the goal of debt reduction can be extremely harmful to the economy. In fact, whenever a state runs a surplus, this never lasts more than a few years (Wray, 1990, 2004), and then recession enters the game to generate new deficits. If public debts have been reduced when compared to the GDP during some period of history, they are nearly always constantly increasing in nominal value, because the total net money of customers needs to go more or less at the same pace as GDP growth plus inflation to satisfy the habits.

However, all our preconceived ideas run counter proper thinking. For instance, in German money is called Geld, a word which is a derivative of gold. It thus carries a meaning that goes beyond the real nature of money where convertibility has been abandoned. On the contrary, debt is called Schuld, the same word that is used for guilt or fault ${ }^{3}$, depending on how we translate. So the naming convention of these basic financial terms may influence German speakers' economic decision-making, since as the Germans pronounce the word debt, they immediately mean what they should do about it, that is getting rid of it, as it is morally bad. By drawing the consolidated graph structure, we are able to replace these linguistic biases with a neutral graph, as only asset/liabilities relations of different types appear.


FIG. 8 Left: Reduction of the public debt from taxation. Right: The should and Schuld points of view on the consolidated public sector (shown in the dashed red circle).

[^3]
## F. Ricardo equivalence against an asymptotic definition of money

In the end, the debate is about what we think the debt will become in the very long run. Indeed, in a system that has abandoned convertibility, central bank money is an eternal debt. In some sense, the central bank owes gold, but it owes it without any deadline, given that it has abandoned convertibility. It would thus be tempting to define money as a debt that is never reimbursed. Interestless money in that definition would thus be just one type of money. As an example, let us consider the paper debts (Demand Notes) that were issued by the US federal government during the Civil War. At that time they were issued as debts, in order to finance the war, but they kept being exchanged and were subsequently considered as currency, without being redeemed into gold, but being used to pay taxes.

The problem with such definition of money is that it is not local in time. Indeed, we need to know the full future of the debt to deduce if it was actually money or if it really was a debt which has been reimbursed. So we have to rely on an imperfect definition, where we would consider that a debt is money if there is a consensus among customers that it will never be reimbursed (or reimbursed by issuing an equivalent debt). This deeply depends on the point of view. For any debt issued by the Treasury, there are thus always two extreme point of views. The first one is held by those who believe that since states always run deficits, it is for sure a type of money. We call this point of view the should point of view since in this point of view we should run deficits, at least in pace with growth and expected inflation. Conversely, for those who think that debts should be repaid, no matter the state of the future economy, the public debt cannot be considered as money. We can call this point of view the Schuld point of view. The debate between should and Schuld is reminiscent of the debate about rational expectations. If the state issues a new debt, does it mean that there is an actual debt of the customers toward the state? Those who think that this debt exists no matter what are in the Schuld camp, whereas those who ignore such possibility, by arguing that people do not look at national accounting for the personal wealth, are in the should camp. For the should point of view, since the state is everybody it is nobody and lies outside. For the Schuld point of view, since the state is nobody, then it means that it is everybody and it lies inside. The Schuld point of view was first formulated formally as the Ricardo-Barro equivalence, according to which taxpayers exactly anticipate future taxes from current deficits. The two points of view are summarized in the right plot of Fig. 8 , the left graph and right graph being respectively the should and Schuld points of view. As already mentioned in § III.A.2, if the Ricardo-Barro equivalence is invoked, the financial wealth of customers and bankers always remains unchanged, even when the Treasury borrows, and remains $M$, that is the total liabilities of the central bank, whereas if the equivalence is ignored (as we think it should be), the financial wealth is increased by public deficits $S$.

It is hard to believe that people's decision-making will be influenced by a possible debt they owe to the Treasury. We think that in order to find a convincing answer in this debate we must look at the past behaviour of major western states. And what we find is that, apart for a few years, they run constant deficits and the sovereign debts keep growing in nominal values. Only on some occasions, some governments manage to run a surplus but this never lasts very long. For the US history, this is certainly the case, where there was just occasional years of surplus in an ocean of deficits (Wray, 2004). The need of running deficits to ensure growth has been advocated by Minksy (1986), and it is best understood from the sectoral financial balances perspective which was popularized as a tool of macroeconomic analysis by Wynne Godley (Godley, 1999, 2000; Godley \& Lavoie, 2007).

## IV. MONETARY AGGREGATES

Having qualitatively described the monetary systems and understood their various mechanisms, it becomes necessary to define monetary aggregates that allow for more quantitative statements on actual monetary systems. All monetary aggregates correspond either to the net money of a consolidated subregion, or to the net financial position between two subregions, that is the net debt from one region to another one. Again, since the subregions are numerous, so are the various monetary aggregates. Using just a few ones to describe the dynamics of the monetary system reflects our interpretation of these systems. All aggregates that we now present are illustrated in Fig. 9 .

- $M_{0}$. The simplest monetary aggregate is $M_{0}$ and is composed of the assets directly held by the customers at the central bank, in the form of coins and paper money. Whenever a customer deposits money at its commercial bank, this amount is reduced, and conversely it is increased when deposits at commercial banks are withdrawn in cash.
- $M_{B}$. If we then include all net deposits held by commercial banks at the central banks, this defines the monetary basis $M_{B}$. Its definition corresponds to the financial position between the central bank and the public sector. By construction, it excludes everything which is related to the public debt issuing in its definition. Hence, the monetary basis will always vary whenever the central bank performs open market operations. Finally note that our definition for $M_{B}$ takes into account only the net deposits of the commercial banks at the central bank, and not just the deposits. This is different from the usual definitions of the monetary basis. With our definition, any central money borrowed at the discount window does not affect the monetary basis $M_{B}$, whereas it would affect the usual definitions of the monetary basis.
- $M_{\text {net }}$. This arbitrariness leads to consolidation of the full aggregate of customers and bankers $M_{\text {net }}$. It counts the Treasury debts as well, as it includes all types of assets entering the bankers net worth. Whenever the central bank engages in open market operations, buying or selling bonds, this monetary aggregate does not change. It corresponds to the net financial assets held by the whole economic system and this is what we have called the net money. An operation of quantitative easing, which is just a massive outright purchase, changes $M_{B}$ but not $M_{\text {net }}$. If we use the ambiguous word money for different aggregates, we would unavoidably disagree on the effect of open market operations. The use of graphs in representing this scenario clarifies the debate.
- $M_{\mathrm{CB}}$. Whenever the Treasury spends by increasing its debt, $M_{\text {net }}$ is increased. This leads to the definition of the central bank aggregate $M_{\mathrm{CB}}$ which encompasses everything but the central bank, and this one will remain constant, whatever the public debt. This aggregate is the reflection of all gold and foreign reserves of the central bank.
- $M_{i}$. We can introduce a further complication if we now decide to exclude the liabilities of customers, but to include only some assets, in an attempt to extend $M_{B}$. This leads to the various definitions $M_{1}, M_{2}, \ldots$, that we gather collectively as $M_{i}$. The more assets of customers are included (with all liabilities excluded in all cases), the larger the aggregate $M_{i}$ is. We thus have a hierarchy $M_{1}<M_{2}<\ldots$ Any loan issued by a bank would surely affect some of the $M_{i}$ as it would increase the assets held by customers. Depending on the nature of the asset, it would count in some $M_{i}$ and not in others. These definitions of the aggregates are a way to estimate the amount of credit made by commercial banks. More
precisely the ratio between the $M_{i}$ and $M_{\text {net }}$ can be used the estimate the amount of credit in the origin of money.


FIG. 9 Schematic representation of $M_{0}, M_{B}, M_{\text {net }}, M_{i}$ and $M_{\mathrm{CB}}$.

## V. FOREIGN CURRENCIES AND EXCHANGE RATES

There are inevitably several sovereign states, each with its own currency, that is its own central bank and its own Treasury, using its own UoA. A complete description of a general monetary system must thus allow for several currencies and their interactions.

## A. Fixed exchange rate system

We first focus on fixed exchange rates systems that were more common in the past. In this section we describe the interactions of various monetary systems, and in the next section we argue clearly why this can never be maintained.

Whenever a national citizen, is paid by a foreign citizen in a foreign currency, he might prefer an asset denominated in his national currency rather than an asset in a foreign currency. In that case, he will ask his bank to exchange it for a national asset as illustrated in the left plot of Fig. 10. The commercial bank will take the foreign currency for itself, and will create a deposit denominated in the national UoA according to the fixed exchange rate. The process might then be repeated between the commercial bank and the central bank as illustrated in the right plot of Fig. 10. The central bank will take the foreign central bank money as an asset and will increase the national central bank money deposit of the commercial bank. As a final result, it is the national central bank which directly possesses the liability of the foreign central bank, that is which has a deposit at the foreign central bank. If the whole procedure is reversed and the customers prefer to hold assets denominated in a foreign currency, this is usually called capital flight.

## B. Fixed exchange rate limitations

The resulting situation is extremely similar to what would have happened if the national citizen had been paid in gold under a regime of convertibility, and if at every stage the gold had been passed


FIG. 10 The foreign currency held be a customer is transferred toward its bank in the left plot and then to the central bank in the right plot.
to the higher level (from the citizen to its bank and then to the central bank). In a fixed exchange rate regime, foreign currencies are treated like gold in a regime of convertibility. If payments are made in gold or in foreign currencies, they can be passed over to the central bank that creates the corresponding net money. Conversely when gold or foreign currencies are sought, the central bank reverts the situation and detaches the asset from its balance sheet, but in the process it also reduces its liabilities in central bank money, effectively reducing the net national money.

As demonstrated in $\S$ II.D, in a regime of convertibility between money and gold, the system cannot hold because of credit creation. The problem is exactly the same in a regime of fixed exchange rate. It is always possible to break down the fixed exchange rate by asking the conversion of national currency into foreign currency, and given that there is more money than net money, there is always a way to reach the point at which the central bank reserves are depleted.

The only possibility for the system to continue would be if the national central bank had the right to borrow foreign central money to the foreign central bank. But then the situation would be rather similar to the case of commercial banks having the right to borrow from a central bank, and this would ensure that all liabilities can be cleared at par, that is with a fixed exchange rate. This was exactly the European situation between 1999 and 2002, when all eurozone currencies had a fixed exchanged rate with the euro foreign currency, and all national central banks were granted the right to borrow from the ECB as much as they needed. Eventually, all national currencies were abandoned and the system was further integrated into a single currency, as this was just a technical intermediary situation. But in general a foreign central bank has no particular interest in granting the right to borrow to the national central bank.

We realize again that granting the right to go in debt is the key to financial power. At every vertex of the tree structure, granting the right to go in debt to vertices which are lower is a form of power on them. The household is afraid that its commercial banker might not grant a loan, and needs to accept the conditions set by the banker, the commercial bank is told at the higher level at which rate, under which conditions, for which maturity, it can borrow central bank money. And if the national central bank now wants to borrow a foreign currency, it will have to accept the conditions of the foreign central bank.

If an economy fixes its currency at parity with a foreign currency, its central bank somehow inserts itself in the tree structure of the foreign country, and instead of being at the apex of the system, it is now situated below the foreign central bank and needs to ask for the permission to go into debt. Even though this type of power of the foreign central bank onto the national central bank is formally similar to the power a commercial bank has on its customers, it is also extremely different. A central bank possesses a form of power on the national financial system as it sets the
condition of debts in commercial banks and thus below for national customers/citizens. At least in principle, it is acting for the interest of the national economy as it is a creature of the state. Instead, when a foreign central bank grants the right to a national bank to borrow foreign central bank money, it has absolutely no reason to act in the interest of the national citizens.

If a weak economy wants to fix its currency to a strong economy, and does not have a full constitutional right to borrow in this external currency, as is always the case, there are essentially two possible situations. Either the country ensures that it will never need foreign currencies to defend the exchange rate. In order to achieve this it would need a strong commercial surplus. Or it needs to borrow the foreign currency, e.g. because of a current account deficit, and needs to comply with the politics imposed by the foreign country. The alternative is thus either to work for free (this is the essence of a permanent commercial surplus for which the dominant foreign countries pays simply by public deficits) or to be told how to work and what to sell (structural reforms imposed by creditors).

## Conclusion

We have argued that graphs for financial stocks are natural tools to describe and discuss the theories of money, because every monetary arrangement is inherently a graph. By connecting together the balance sheets in a graph rather than explicitly writing one after another the balance sheets of each institution [e.g. in the figures of McLeay et al. (2014) or in the balance sheets explanations of Keen (2014)], we found that a hierarchical nature expressly appears. We advocate using graphs whenever monetary systems are discussed, as they allow for a more visual and direct representation in which consolidations are made straightforward.

We have intentionally restricted our description to asset/liability relations, ignoring real assets. We also ignored the representation of private companies, but they can easily be incorporated as a set of financial assets, real assets and liabilities, which induce a net worth possessed by the owners. Instead we focused on how these tools can be used to analyze the structure of the state, with its central bank and its Treasury, and understand the nature of money. We argued that in sovereign states running sovereign currencies, the Treasury and the central bank can be meaningfully consolidated, as we showed how their actions are coordinated thanks to monetary policy. We explained that the state only exogenously controls the net money through public deficits, and sets the boundary conditions for the financial structure of the private sector, but it does not control the amount of credit which is endogenously determined. Only the salient features have been illustrated with graphs, but the reader is encouraged to draw the graphs corresponding to any discussion in order to develop graphical representations of monetary systems.

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[^1]:    ${ }^{1}$ More realistically the physical group of persons made of stockholders. But throughout this paper we symbolically describe these as the banker.

[^2]:    ${ }^{2}$ The case of the European Central Bank is more complicated since it is not allowed to buy T-bonds in normal times, but it has engaged recently in such programs to stabilize the eurozone.

[^3]:    ${ }^{3}$ As noticed and analyzed in Graeber (2011), this is also the case in many ancient languages (Sanskrit, Hebrew, Aramaic).

