Predictions on the application of the Hanle effect
to map the surface magnetic field of Jupiter

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Abstract

In this paper we evaluate the possibility of detecting, for the first time, the surface magnetic field of Jupiter (∼1 bar level) by observing the change of linear polarization induced by the Hanle effect on the H Lyman-alpha (Lyα) emission line of the planet. We find that, indeed, the Hanle effect, which results from the interaction between a local magnetic field and the atomic polarization induced by absorption of anisotropic radiation, is sensitive to relatively weak values of the strength of the magnetic fields expected on planets. First, we show that for the Lyα emission backscattered by atomic H in the presence of a magnetic field, the Hanle effect is polarizing. This new result is in total contrast to the depolarizing effect predicted and observed for emission lines scattered at right angles in solar prominences. Additionally, to estimate the polarization rate for the case of Jupiter, we have considered three magnetic field models: a dipole field for reference, an O₄ based model [Connerney, J.E.P., 1981. The magnetic field of Jupiter—A generalized inverse approach. J. Geophys. Res. 86, 7679–7693], and finally, an O₆ based model [Khurana, K.K., 1997. Euler potential models of Jupiter’s magnetospheric field. J. Geophys. Res. 102, 11295–11306]. In all models, we show that for the jovian backscattered Lyα line, the Hanle effect does enhance the Lyα linear polarization; the polarization rate may exceed 2% at specific regions of the jovian disc, making detection possible either remotely or from an orbiter around Jupiter. In general, depending on the instrumental sensitivity and the observing strategy used, we show that accurate mapping of the linear polarization rate at the planetary surface (thermosphere) or off-disc (corona) may provide a rather accurate estimate of the jovian total magnetic field strength on large area scales.

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1. Introduction

Electrodynamic effects play a global and significant role in the state and energization of the Earth’s ionosphere/magnetosphere (e.g., Schunk and Nagy, 2000). On Jupiter, the average auroral processes have an input power about four orders of magnitude larger than on Earth; in addition, the jovian magnetosphere is influenced by a strong internal source of plasma from the Io torus and by the fast planetary rotation. Electrodynamic effects are thus expected to play a significant role, particularly in the state and energization of the ionosphere–corona–plasmasphere system of the planet, including the satellites Io and its plasma torus, Ganymede, and Europa. This environment can be very dynamic and strongly dependent on a complex network of currents that flow in the magnetosphere, connecting different regions on large scales. Field aligned currents are the best known as they connect via magnetic field flux tubes, satellites, middle and tail magnetospheric regions to the planet’s polar regions, leading to stunning auroral emissions including satellites footprints.
It follows that the interplay (through waves, currents, etc.) between electromagnetic fields and particles from different sources makes any diagnostic used to understand the system possible only if nearly simultaneous information on the local plasma conditions and fields’ network topology on large scales are obtained, particularly in the plasmasphere region where most of the icy satellites orbit.

In this system, the magnetic field distribution plays a key role in shaping the particles content and extension of the magnetospheric cavity. Theoretically, the plasma environment of the planet could be studied consistently using either MHD, kinetic, or hybrid approaches (Winske and Omidi, 1996). Unfortunately, sophisticated codes are required, thereby making the derivation of any diagnostic concerning the system a difficult task. Another approach is to build theoretical models of the magnetic field, which is represented as the sum of an internal and induced magnetospheric components. These models are usually validated through direct comparison with in situ data obtained by the different spacecraft that have visited the jovian system so far (Pioneers, Voyagers, Ulysses, Galileo, Cassini) and sampled different locations of the magnetosphere. Such field models are usually used as a key input to more sophisticated studies, including the mapping of field lines between different regions of the magnetosphere and the planetary ionosphere (aurora, local time effects, etc.), plasma dynamics, particles dynamics, and so forth. A dipolar field configuration would simplify these studies, particularly for linking auroral emission features to related magnetospheric regions. Far from that idealistic configuration, the jovian field is in fact dominated by strong quadrupolar and octopolar terms. These field components, which are two to three times greater than the dipolar term in the atmosphere, fall faster than the dipole field when they are far from the planet, diminishing to near the detection threshold at the orbital distances typically sampled during spacecraft encounters. Since no observational data exist for distances from the planet smaller than \( \sim 1.6 R_J \), the non-dipolar component of the field remains unconstrained, thus casting uncertainties on all other related studies of the magnetosphere.

One way around this circular problem would be to obtain a direct measurement of the magnetic field near the planetary surface in a region defined as a few thousands kilometers layer above the visible reference level at \( \sim R_J \). As we show herein, such a measurement is feasible via resonance line polarimetry using the Hanle effect. The brightest jovian emission line in the UV is the atomic hydrogen Ly\( \alpha \) line, which is either solar line backscattered by the thermospheric hydrogen or collisional induced auroral lines at high latitudes. Fortunately, the Hanle effect induced in the Ly\( \alpha \) emission polarization rate is sensitive to magnetic field strengths comparable to the magnitude of the jovian field nearby the planet (which may exceed \( \sim 14 \) Gauss in some locations). Observation of the polarization of the Ly\( \alpha \) emission line should therefore map the strength of the magnetic field at different altitudes of the atmosphere and corona and at different latitudes/longitudes. In the following, we introduce the Hanle effect, then focus on the case of the Ly\( \alpha \) resonance line of Jupiter. The polarization rate is deduced using classical and quantum mechanical formalisms developed by Bommier (1997, 1980) and Bommier and Sahal-Bréchot (1978, 1982). Diagrams are then derived showing the complex relationship between polarization, magnetic field strength and scattering geometry in general. Three models of the jovian magnetic field are then considered: a dipolar field (for reference), an \( O_4 \) based model (Connerney, 1981) and an \( O_5 \) based model (Khurana, 1997). Predictions of polarization rates for different viewing geometry are thus proposed for the different magnetic field models. Instrumental options and perspectives for future spectro-polarimetry techniques to study the jovian magnetic field are then proposed for either Earth or Jupiter based missions.

2. The Hanle effect of the Ly\( \alpha \) line

The Hanle effect is commonly understood as the depolarization and rotation of linear polarization direction, due to an ambient magnetic field, of a line linearly polarized by radiative scattering. For scattering to be a polarizing mechanism, the incident radiation needs to be anisotropic. The physical origin of the Hanle depolarization lies in a competition between the natural damping and the Larmor precession, where the latter process is also responsible for the rotation of the polarization direction. When both effects are of the same order of magnitude—i.e., when \( \omega_L \tau \sim 1 \), where \( \omega_L \) is the Larmor pulsation that is proportional to the field strength (1.4 MHz/Gauss, ignoring the Landé factors) and \( \tau \) is the upper level lifetime—the effect is sensitive mainly to the magnetic field strength. When the depolarization is complete—i.e., when \( \omega_L \tau \gg 1 \) —only the sensitivity to the field direction remains. In the laboratory, the effect has been used to measure lifetimes, the magnetic field being known. Yet in Astrophysics, the effect has been used as a magnetic field diagnostic from known lifetimes. The first astrophysical detection of the Hanle effect was made from solar prominences (right angle scattering of the photospheric radiation), leading to field vector diagnostics, with an average field strength of 6 Gauss (see a detailed example in Bommier et al., 1994, and references therein). For the case of \( \omega_L \tau \sim 1 \), the magnetic field sensitivity is a function of the lifetime of the transition, so that in general, \( \sim 10 \) Gauss fields can be detected by using visible resonant lines (\( \tau \sim 10^{-8} \) s), and the UV lines can provide sensitivity to even higher field strengths (\( \sim 40 \) Gauss for hydrogen Ly\( \alpha \)). On the other hand, the forbidden lines of the solar corona are sensitive to the coronal field direction only, due to their very long lifetimes (see, for instance, Sahal-Bréchot, 1974, 1977). The magnetic field sensitivity depends thus on the details of the line under study. For additional examples, the reader is referred to a table of line magnetic sensitivities in Sahal-Bréchot (1981).
Another aspect of the Hanle effect is its highly non-linear effect, allowing the detection of magnetic regions embedded within non-magnetic regions, for instance, or even mixed-direction fields in unresolved sources (Ignace et al., 2004).

The quantum mechanical description of the Hanle effect is the modification, owing to the magnetic field, of the so-called “coherences” (or phase relationships between sub-level’s wavefunctions) created in the atomic upper level by absorption of anisotropic radiation. The atomic density matrix is a formalism well suited to the description of these coherences in a natural medium (i.e., a “non-pure” case). The proper evaluation requires the balance of all the polarizing and depolarizing mechanisms acting on the scattering atom. Such a balance is realized by solving the statistical equilibrium equations for the atomic density matrix elements, as done below.

The novelty in the present study is the consideration of photons backscattering rather than the usual right angle scattering. In such a new scattering scheme, the Hanle effect features are completely different: for an anisotropic incident radiation, the backscattered radiation is unpolarized in the absence of a magnetic field, yet the effect of a magnetic field is now polarizing, especially when the field lies in the plane of the sky, perpendicular to the line-of-sight. The linear polarization direction is aligned with the projection of the field on the plane of the sky. In the present work we investigate the order of magnitude of this effect in the Jupiter Lyα case, and compare it with the expected detector sensitivity in order to evaluate the feasibility of a Hanle diagnostic of the jovian magnetic field.

In atomic polarization studies, the depolarizing effect due to hyperfine structure could be important in most cases. Such is not the case for the hydrogen Lyα, as shown by Bommier and Sahal-Bréchot (1982). In this instance, although the hyperfine splittings are not much smaller than the natural width, they are sufficiently small indeed so that the depolarization is unimportant. As a consequence, the hyperfine structure is ignored in the following treatment.

Another mechanism able to modify the anisotropy of the incident radiation is the radiative transfer (RT). In the presence of a magnetic field, partial redistribution of polarized radiation is known to be a complex problem (Bommier, 1997b). When the ambient magnetic field is weak, it was shown that while the Hanle rotation effect is sensitive to photons scattering in the line core and may even vanish in the far wing, the Hanle polarizing/depolarizing effect remains almost unsensitive to scattering, particularly in the line wing (Bommier, 1997b; Stenflo, 1998). In the absence of a magnetic field, the frequency redistribution mechanism is known to broaden the core of the jovian Lyα line while the line wing remains almost unchanged (Ben Jaffel et al., 1988). Also, for polarimetry diagnostics, we expect each successive scattering event to further randomize the phase geometry of the photon field, thereby reducing the overall level of polarization, a tendency in opposition to the polarization enhancement induced by the Hanle effect. Multiple scatterings therefore introduce uncertainty into the estimate of the Hanle effect because of the contribution to the total emission of the optically thick depolarized Lyα line core. Moreover, during a scattering event, frequency redistribution may bring photons from a spectral range to another, particularly from the line core to the wing. Of course a frequency transition range does exist between the two spectral ranges. According to our theoretical estimate from the exact RT redistribution function, and based on Lyα emission line observations made by HST/STIS at high resolution (Emerich et al., 2003; Kim, Ben-Jaffel, and Clarke, in preparation), we estimate that, beyond four Doppler unit from the line center, the weight of redistributed photons to the total brightness is small. For that spectral range, the impact on the polarization rate is negligible as the ratio of the redistributed photons from the entire line core into the wing should not exceed 4% of the total line wing signal. For reference, the wing signal in that case represents ~44% of the total jovian line. If one defines the wing as the frequency range beyond three Doppler units from the line center, then the contamination will not exceed 6% and the wing signal should represent almost ~51% of the total jovian line. In analyzing the line profile emitted by the optically thick disc, we thus focus our study on the wing of the Lyα line for which the RT effects are negligible. This assumption should affect neither our results nor our observational strategy because the line wing brightness is known to be strong and is dominant over the line’s core near the planetary limbs (Ben Jaffel et al., 1993; Emerich et al., 1996). Off-disc, as shown by some recent HST/STIS long slit observations, the jovian Lyα line is optically thin in the coronal region, for altitudes starting ~3000 km above the planetary visible limb (Emerich et al., 2003; Ben Jaffel et al., in preparation). For that outer layers, the Lyα brightness is still strong and the entire line should be included in the diagnostic of the Hanle effect without any restriction on the frequency range.

In summary, to avoid the depolarizing effect of multiple scattering, the Hanle effect diagnostic could be unambiguously evaluated either from the wing of the Lyα line emitted by the optically thick planetary disc or from the full Lyα line emitted by the jovian optically thin corona, ~3000 km above the visible limb. Of course, one way to avoid these limitations is to solve the coupled statistical equilibrium and RT equations for the entire line, a complex task that is out of the scope of the present exploratory study.

2.1. Theoretical background

In the following, we consider a Cartesian reference frame \(Oxyz\) tied to the target (Jupiter or any planet) where \(Oz\) represents the polar axis. In this frame, the \(\vec{\Omega}\) direction vectors that appear in the following calculations are defined by two angles: the polar angle \((0^\circ < \theta < 180^\circ)\) and the azimuth angle \((-180^\circ < \chi < 180^\circ)\), as displayed in Fig. 1. We denote as \(\vec{\Omega}(\theta_0, \chi_0)\) the Sun’s direction. The incident
solar radiation propagates along the direction $-\vec{\Omega}_0$. Since the formalism of atomic scattering is invariant to the radiation propagating in a positive or negative direction, one may consider as well that the incident radiation field propagates along the $+\vec{\Omega}_0$ direction. We also denote as $\vec{\Omega}_B(\theta_B, \chi_B)$ the magnetic field direction, and $\vec{\Omega}_L(\theta_L, \chi_L)$ the line of sight direction to the observer.

The polarization rate is derived directly from the solution of the statistical equilibrium equations that describe the atomic level populations excited by anisotropic Lyα photons. We use the standard formalism of density matrix that describes the atomic level polarization, together with the phase matrix formalism that allows connection of the scattered and incident radiation properties (Bommier, 1997a, 1997b; Bommier and Sahal-Bréchot, 1978; Landi Degl’Innocenti, 1983, 1984).

The Lyα line connects the hydrogen atom 2p level to the fundamental 1s level. Due to spin–orbit interaction, the 2p level splits, by coupling of the orbital ($L = 1$) and spin ($S = 1/2$) kinetic momenta, into the 2p$_{1/2}$ and 2p$_{3/2}$ fine structure levels. The lower 1s level has only one fine structure level 1s$_{1/2}$. The resulting structure is given in Fig. 2. It follows that the Lyα line has two components.

In the 2p$_{1/2} \rightarrow$ 1s$_{1/2}$ Lyα$_1$ component, the kinetic momentum of the upper level is 1/2, and light emitted in this transition is unpolarized.

In the 2p$_{3/2} \rightarrow$ 1s$_{1/2}$ Lyα$_2$ component, the kinetic momentum of the upper level is 3/2, and light emitted in this transition may be linearly polarized.

Both transitions have the same spontaneous deexcitation rate $A = 6.265 \times 10^8$ s$^{-1}$. We outline below the main steps of the theoretical formalism.

2.2. Statistical equilibrium energy equations

We denote as $\rho^K_0$ the density matrix element for the upper level $b$ in the tensorial component $(K, Q)$, and $\rho^K_0$ the density matrix element for the ground level $a$, which is unpolarizable ($J = 1/2$), so that it has a single even tensorial component ($K = 0$, $Q = 0$). As the incident radiation has no component of odd tensorial order (the so-called “orientation”), the density matrix even elements (the so-called “alignment”) are coupled together independently of the odd elements. This results in canceling odd order components that cannot be excited. Since the incident radiation does not carry orientation, atoms behave accordingly. The statistical equilibrium equation for level $b$ can thus be written as

$$\frac{d}{dr} (\sqrt{(2J'+1)} b b \rho^K_0) = 0 = -i \omega_L g' Q \sqrt{(2J'+1)} b b \rho^K_0 + B_{ab} \sqrt{(2J'+1)} a a \rho^K_0 (-1)^Q \bar{w}^{(K)}_{J'J} \times \int \frac{d\vec{\Omega}}{4\pi} T^K_Q(0, \vec{\Omega}) I(\vec{\Omega}) - A_{ba} \sqrt{(2J'+1)} b b \rho^K_0,$$  \hspace{1cm} (1)

where $J'$ is the kinetic momentum of upper level $b$, $J$ is the kinetic momentum of lower level $a$, $\omega_L$ is the Larmor pulsation: $\omega_L = -\mu_B B / \hbar$, where $\mu_B$ is the Bohr magneton: $\mu_B = g \hbar / (2m_e)$; $g_J'$ is the Landé factor of the upper level $J'$: $g_J' = (3/2) + (S'(S' + 1) - L'(L' + 1))/(2J'(J' + 1))$, $S'$ and $L'$ being respectively the spin and orbital quantum numbers corresponding to the upper level $J'$. $B_{ab}$ and $A_{ba}$ are the Einstein coefficients for absorption and spontaneous emission, respectively. $T^K_Q(i, \vec{\Omega})$ is the spherical tensor for polarimetry (Landi Degl’Innocenti, 1984), where $i$ is the index of the Stokes parameter ($i = 0, 1, 2, 3$ for $I, Q, U, V$, respectively), and $\vec{\Omega}$ is a given direction. Here, this tensor is applied to the incident solar radiation. We assume that this radiation is unpolarized. Its Stokes parameters, thus, are:
\[ I = I(\tilde{\Omega}_L') \] and \( Q = U = V = 0 \). We further assume that the incident solar radiation is unidirectional, so that it can be represented by a Dirac angular function \( \delta(\tilde{\Omega}_L') \)

\[ I(\tilde{\Omega}) = I_0 \delta(\tilde{\Omega}_L'). \]  

(2)

As in Bommier (1997b), \( T^K_Q(i, \tilde{\Omega}) \) has to be expressed by using the two rotations from the magnetic field reference frame in which the tensorial orders are defined, to the jovian reference frame and then to the incident radiation reference frame. The final expression of \( T^K_Q(i, \tilde{\Omega}) \) depends on both the incident radiation direction \( \Omega_0 \) and the magnetic field direction \( \tilde{\Omega}_B \) in the jovian reference frame. Accordingly, we denote these coefficients as \( T^K_Q(i, \tilde{\Omega}_0, \tilde{\Omega}_B) \) in the following treatment. We rewrite the statistical equilibrium equation for single scattering thusly:

\[
0 = -i \omega g_{ij} Q \sqrt{(2J_i + 1)} \frac{\alpha_{ij}}{\rho^K_Q} + B_{ab} \sqrt{(2J_i + 1)} \frac{\alpha_{ij}}{\rho^K_Q} w_{ij}^{(K)}(\Omega) T^K_Q(0, \tilde{\Omega}_0, \tilde{\Omega}_B) \frac{I_0}{4\pi} - A_{ab} \sqrt{(2J_i + 1)} \frac{\alpha_{ij}}{\rho^K_Q},
\]

(3)

where \( w_{ij}^{(K)} \) is a set of coefficients that characterize the atomic polarizability and that are defined as follows (Landi Degl’Innocenti, 1984):

\[
w_{ij}^{(K)} = \begin{cases} 1 & J' = J \\ 0 & J' \neq J \end{cases}
\]

(4)

\[
w_{ij}^{(K)} = \begin{cases} 1 & J' = J \\ 0 & J' \neq J \end{cases}
\]

(5)

where \( \{ \} \) is a 6-j symbol. Note that \( w_{ij}^{(0)} = 1 \). See a table of \( w_{ij}^{(K)} \) coefficients in Landi Degl’Innocenti (1984).

2.3. Resolution of the statistical equilibrium equations

By setting \( \sqrt{(2J_i + 1)} \frac{\alpha_{ij}}{\rho^K_Q} = 1 \), one can solve Eq. (3):

\[
\sqrt{(2J_i + 1)} \frac{\alpha_{ij}}{\rho^K_Q} = \frac{B_{ab}}{A_{ab} + i \omega g_{ij} Q} w_{ij}^{(K)} T^K_Q(0, \tilde{\Omega}_0, \tilde{\Omega}_B) \frac{I_0}{4\pi}.
\]

(6)

From the upper level density matrix elements, we derive the Stokes parameters of the emitted radiation as the emissivity integrated over the line profile (Landi Degl’Innocenti, 1983):

\[
\epsilon_j = \sum_{KQ} \sqrt{(2J_i + 1)} \frac{\alpha_{ij}}{\rho^K_Q} T^K_Q(j, \tilde{\Omega}_L') \frac{I_0}{4\pi}.
\]

where \( \epsilon_0 = T, \epsilon_1 = Q, \epsilon_2 = U, \epsilon_3 = V \). \( \tilde{\Omega}_L' \) is the line of sight direction expressed in the magnetic field reference frame. These coefficients are expressed here in \( s^{-1} \). To recover the line-integrated emissivity in unit of \( (\text{erg/cm}^2/\text{s}) \), one has to multiply them by the constant \( N \frac{h\nu}{4\pi} \), where \( N \) is the lower level population density in \( \text{cm}^{-3} \), and \( \nu \) is the line frequency.

\[ T^K_Q(j, \tilde{\Omega}_L') \] has now to be expressed in the jovian reference frame, where \( \tilde{\Omega}_L \) is the line of sight direction, and \( \tilde{\Omega}_B \) the direction of the magnetic field, thus yielding:

\[
\epsilon_j = \sum_{KQ} \sqrt{(2J_i + 1)} \frac{\alpha_{ij}}{\rho^K_Q} T^K_Q(j, \tilde{\Omega}_L, \tilde{\Omega}_B).
\]

Transposing back the density matrix solution given by Eq. (6) into the emissivity expression, one can show that for any atomic level \( b \):

\[
\epsilon_j = \sum_{KQ} \frac{B_{ab}}{A_{ab} + i \omega g_{ij} Q} \frac{A_{13}}{4\pi} \frac{I_0}{4\pi} (w_{ij}^{(K)})^2
\]

\[
\times (-1)^Q T^Q_{-Q}(0, \tilde{\Omega}_0, \tilde{\Omega}_B) T^K_Q(j, \tilde{\Omega}_L, \tilde{\Omega}_B).
\]

In the case of the Ly\( \alpha \) line, the two upper levels \( 2p_{1/2} \) and \( 2p_{3/2} \) are referenced below to numbers 2 and 3, respectively. For level 2, only the \( K = 0 \) component exists. For level 3, two components \( K = 0 \) and \( K = 2 \) contribute. By adding the contributions from the two upper levels, we obtain

\[
\epsilon_j = B_{12} I_0 \delta_{j,0} + \sum_{KQ} B_{13} \frac{A_{13}}{4\pi} \frac{I_0}{4\pi} (w_{j}^{(K)})^2
\]

\[
\times (-1)^Q T^Q_{-Q}(0, \tilde{\Omega}_0, \tilde{\Omega}_B) T^K_Q(j, \tilde{\Omega}_L, \tilde{\Omega}_B),
\]

(8)

where

\[
B_{12} = A_{21} \frac{c^2}{2h\nu^3}
\]

and

\[
B_{13} = 2A_{31} \frac{c^2}{2h\nu^3}.
\]

Since \( A_{21} = A_{31} = A \), and since the Stokes parameters appear only as ratios \( Q/I, U/I, V/I \) in the polarization rate, one can cancel the factor \( A c^2/(2h\nu^3) I_0(4\pi) \) which is common to all contributions, and obtain thus in reduced units:

\[
\tilde{\epsilon}_j = \delta_{j,0} + \sum_{KQ} \frac{2A}{A + i \omega g_{ij} Q} (-1)^Q (w_{j}^{(K)})^2
\]

\[
\times T^Q_{-Q}(0, \tilde{\Omega}_0, \tilde{\Omega}_B) T^K_Q(j, \tilde{\Omega}_L, \tilde{\Omega}_B).
\]

(11)

The normalized emissivity is only valid when no proper emission, like the auroral emission, adds to the total signal. This equation is used below to calculate the Ly\( \alpha \) line polarization rate.

2.4. Analytical solution for the Hanle effect on the Ly\( \alpha \) line and test of the numerical code

In order to check the results of the computer FORTRAN code used to calculate the polarization rate, as well as for the sake of clarity, we consider an analytical solution for the Hanle effect (Landi Degl’Innocenti and Landi Degl’Innocenti, 1988). We consider the particular case of photon backscattering and a magnetic field lying on the plane of the sky (a simple geometry, whereas our numerical code is able to treat any geometry). This process corresponds to the
jovian Ly\(\alpha\) reflected emission. If the magnetic field vector is parallel to \(Oz\), which is also the “positive \(Q\) direction” (or reference direction) for the Stokes parameters, and with the line-of-sight along \(Ox\), by applying the analytical formalism of Landi Degl’Innocenti and Landi Degl’Innocenti (1988) to the backscattering case of an unpolarized pencil of incident radiation, and taking into account that the scattered Ly\(\alpha_2\) intensity is twice the one of the unpolarized Ly\(\alpha_1\) for a frequency-flat incident radiation, one obtains for the Ly\(\alpha\) line:

\[
\frac{Q}{T} = \frac{3\gamma^2}{7 + 25\gamma^2},
\]

where

\[
\gamma = \frac{\omega_{4f}}{A},
\]

where the Einstein coefficient for spontaneous emission from the Ly\(\alpha\) level 2 is \(A = 6.265 \times 10^8 \text{ s}^{-1}\). Assuming \(B = 10\) Gauss, one has

\[
\gamma = 0.18716,
\]

so that

\[
\frac{Q}{T} = 1.33426851 \times 10^{-2}.
\]

For a magnetic field direction perpendicular to the line of sight, but not necessarily parallel to \(Oz\), we obtain a similar value with our numerical code

\[
p = \frac{Q}{T} = 1.3343 \times 10^{-2}.
\]

The polarization direction is found parallel to the magnetic field in backscattering when the field is perpendicular to the line-of-sight. In addition, we have checked that for strong magnetic fields, those larger than 400 Gauss, the polarization rate tends toward the theoretical value \(3/25 = 0.12\). Our numerical code derived the same limiting value of 0.12.

Furthermore, the analytical expression applied to 90° scattering without a magnetic field leads to a linear polarization degree \(p = 3/11 = 0.2727\). Moreover, if only the Ly\(\alpha_2\) component \(2p_{3/2} \to 1s_{1/2}\) is considered, the theoretical expectation is \(p = 3/7 = 0.4286\). Due to identity of quantum numbers, these values are the same as for the sodium D lines that were derived by Mitchell and Zemansky (1934), when a frequency-flat incident radiation is assumed (i.e., same incident intensity in D1 and D2). These analytical results have also been used to test our numerical code. Finally, numerical results of Bommier and Sahal-Bréchot (1982), in this 90° scattering geometry but with a magnetic field, were also used for comparison and testing of the code.

In the next section, we apply the numerical code to the particular case of the jovian backscattered Ly\(\alpha\) emission line in the presence of a relatively weak magnetic field.

3. Numerical model

3.1. Polarization rate calculation

The linear polarization rate is defined as:

\[
p = \sqrt{(Q/I)^2 + (U/I)^2}.
\]

The direction of linear polarization, referred to by the angle \(\alpha\) with respect to \(Oz\), is defined as:

\[
\cos 2\alpha = \frac{Q}{\sqrt{Q^2 + U^2}},
\]

\[
\sin 2\alpha = \frac{U}{\sqrt{Q^2 + U^2}}.
\]

Our numerical code requires the direction and strength of the magnetic field, and, making use of Eq. (11), provides the polarization rate, the direction of polarization and the ratios \(Q/I\) and \(U/I\).

3.2. Results

At this point, one may question whether the Hanle effect on the polarization of the Ly\(\alpha\) line is adequate to derive the jovian magnetic field. In order to answer this question definitively, we have plotted the backscattering polarization rate versus the angle \(\theta_B\) between the magnetic field and the plane of the sky for a magnetic field strength of 10 Gauss (Fig. 3). For a magnetic field direction perpendicular to the incident light corresponding to \(\theta_B = 0\), we derive a maximum polarization rate of 1.3343 \times 10^{-2}, large enough for a detection by modern spectro-polarimeters operating in the FUV range (Harris et al., 2003). By contrast, for \(\theta_B = 90\), the polarization rate goes to zero, showing that when the field

![Fig. 3.](image-url)
magnetic orientation is symmetrical with the radiation field, the Hanle effect vanishes even though the incident radiation is anisotropic. As we shall see in the following sections, the jovian surface magnetic field is not symmetrically oriented with the incident solar radiation field and could be larger than 10 Gauss, which makes detection of the Hanle effect on the jovian Lyα line polarization quite possible from space.

We have also considered the sensitivity of the polarization rate as a function of the phase angle (the angle made by Jupiter, Earth, and the Sun), an angle that cannot exceed 11°–12° for Jupiter. Taking $\chi_L = 11^\circ$ (angle between the incident light direction and the line of sight) and $B = 10$ Gauss, we obtain a polarization rate $p = 6.8797 \times 10^{-3}$, smaller than the rate obtained with a zero phase angle (alignment between Jupiter, Earth, and the Sun). For remote observation from Earth, one should favor observational conditions for Earth near opposition for the best chance to detect the Hanle effect. This orbital condition may last more than two months each year for a phase angle not exceeding $\pm 5^\circ$ that is most favorable for the Hanle effect detection. In order to quantify the sensitivity of the polarization rate to the planetary magnetic field and geometry of observation, we present in the following section an in-depth parametric study in the context of the jovian system that should help us build new observational techniques to deduce the jovian magnetic field on large area scales.

4. Parametric study of the Lyα polarization

To evaluate the dependency of the Lyα polarization rate on the different parameters that appear in its expression Eq. (11), we consider sophisticated diagrams that provide the rate and direction of the polarization versus the magnetic field strength and orientation angle $\theta_B$ (only $\chi_B = +90^\circ$ and $-90^\circ$ are considered). We consider two extreme cases: 90° scattering (typical of solar observation), and backscattering that applies for planets. These diagrams could be used either to derive the field strength and one parameter of the direction of the magnetic field if the polarization rate and another parameter of the field direction are known or vice versa. To have a full description of the ambient magnetic field, one needs, optimally, two emission lines of different sensitivity ($\omega_L, \tau$).

For the sake of clarity, we will consider below two approaches: an observational approach that provides the polarization rate $p$ versus the polarization direction, and a theoretical approach that uses the $Q/I$ and $U/I$ coordinates (Figs. 4 and 5).

In the case of scattering at 90°, Fig. 4 shows the depolarizing effect along a given $\Psi_B$ iso-contour with increasing magnetic field strength, and the increase in rotation of the polarization from $\Psi_B = 0$ to $\Psi_B = 45$. Because of the fundamental degeneracy, the curves $\Psi_B = 45^\circ$ and $\Psi_B = -45^\circ$ are exactly the same. By contrast, as shown in Fig. 5, the polarizing effect of the magnetic field in the backscattering process increases the polarization rate with the magnetic field strength. This new theoretical result is very important because it stresses that for reflected radiation from planetary atmospheres, the Hanle effect enhances the Lyα polarization rate and thus favors its detection. In the following section, we consider realistic magnetic field models of Jupiter and show how the Lyα polarization rate is enhanced and detectable over those regions of the jovian disc where the magnetic field is strongest.

![Fig. 4. Scattering at 90°: The incident radiation is a pencil backward along $Ox$, and the line-of-sight is taken along $Oy$. The magnetic field lies in the $yOz$ plane. $\Psi_B$ is the angle between the magnetic field and the line-of-sight, oriented by the incident radiation direction. Accordingly, $\Psi_B = 90 - \chi_B$. Left: Diagram showing the polarization rate $p$ versus polarization direction for different values of, respectively, the magnetic field strength and the angle $\Psi_B$. Right: $Q/I$ versus $U/I$. This diagram is equivalent to the diagram on the left panel but mostly oriented to theoretical rather than to observational applications.](image-url)
5. Application to Jupiter

5.1. General assumptions and graphical representation

To visualize the magnetic field and the associated polarization distribution over the planetary sphere, we considered two families of maps. The first mapping corresponds to observations made at different central meridian longitudes (CML) as the planet rotates, while the second technique maps on the apparent disc for a fixed CML (i.e., observable hemisphere). We include the planetary orbital inclination only in those maps associated with the apparent planetary disc. In this case, the planetary axis tilt is applied in the plane at System III longitude 200°.

In all cases, the model takes into account the center-to-limb effect of the apparent planetary disc. For the opacity calculations, we know that the huge gravity of Jupiter makes its atmosphere well stratified. The atomic hydrogen distribution, which is controlled by the hydrocarbons chemistry and vertical transport, shows a density peak nearby the homopause level and its abundance may dominate all other constituents in the upper thermosphere (Strobel, 1974). Consequently, the Hanle effect diagnostic, using the line wing, should not be very sensitive to the height variation of the jovian magnetic field because the line wing source function is confined to a thin layer around the homopause level. By contrast, in the jovian corona, for altitudes larger than ~3000 km above the visible limb, the Hanle diagnostic should be sensitive to the height variation of the field strength because the corresponding source function is spatially extended. In any realistic analysis of future observations, one has therefore to account for the height variation of the strength of the magnetic field. Yet, in this exploratory study we assume that the field is not height dependent with a strength that corresponds to the averaged field in the probed layers. One must then integrate the emitted radiation field along the line of sight, taking into account the apparent opacity that is larger along the limb slant path than at disc center. Practically, assuming an optically thin approximation, the Stokes parameters should be multiplied by the optical thickness of the emitting region. However, since the Stokes parameters appear only as ratios in the polarization rate, the opacity has actually no effect in the optically thin approximation. This approximation is valid both for the wing of the jovian Lyα emission line that originates from the planetary disc and for the full line from the jovian corona. Note that the line wing spectral range represents most of the planetary brightness near the sunlit limb (Ben Jaffel et al., 1993, 1995). In the ideal case, one has to include multiple scattering effects, particularly in the line core, by solving the radiative transfer equation for the four Stokes parameters, a task that is well beyond the prediction scope of the present study.

5.2. Magnetic field models of Jupiter

In this section, we consider three models of the magnetic field of Jupiter, namely: a dipolar field model; a second model, defined as the sum of the $O_4$ internal field and a current sheet component (Connerney, 1981); and the so-called Euler potential model, defined as the sum of the $O_6$ internal field component (Connerney, 1993) and an external field represented by Euler potentials (Khurana, 1997). Other models are proposed in the literature but should not deviate much from the models selected in our exploratory work. The dipolar model is considered here to provide a simple reference map of the magnitude of the jovian Lyα
polarization rate distribution over the planet. More realistic models are the $O_4$ based model and the $O_6$ based model, which are defined from in situ observations made by one or several spacecraft that have visited the jovian system (Pioneers, Voyagers, Galileo, etc.). Unfortunately, most of the data so far obtained have been gathered for regions relatively far from the planet (e.g., Khurana, 1997). The magnetic field distribution close to the planetary ionosphere is almost unknown and relies mostly on extrapolation models that “fit” the scattered observations obtained at distances larger than $\sim 2 R_j$ from the planet. Other sources of information on the jovian magnetic field rely on auroral emissions, particularly satellites’ footprints (Connerney et al., 1998; Clarke et al., 2002), but the connection is neither straightforward nor self-consistent. Note also that the measured jovian magnetic field has shown variability between all the spacecraft encounters. For example, the polar magnetic field measured by Ulysses was significantly weaker than measured by previous spacecraft.

5.2.1. Dipole field model

Starting from the Cartesian coordinates of a dipolar magnetic field, we can deduce the direction $(\chi_B, \theta_B)$ and the strength of the magnetic field (Kivelson and Russell, 1995).

\begin{align*}
B_x &= 3xz M_r r^{-5}, \\
B_y &= 3yz M_r r^{-5}, \\
B_z &= (3z^2 - r^2) M_r r^{-5}, \\
x &= r \sin \theta \cos \phi, \\
y &= r \sin \theta \sin \phi, \\
z &= r \cos \theta, \\
r &= R_j = 71492 \times 10^3 \text{ m}, \\
\chi_B &= \frac{180}{\pi} \tan \frac{B_y}{B_x}, \\
\theta_B &= \frac{180}{\pi} \tan \sqrt{\frac{B_x^2 + B_y^2}{B_z}}, \\
B &= \sqrt{B_x^2 + B_y^2 + B_z^2},
\end{align*}

where $M_r \sim 1.53 \times 10^{30} \text{ Gauss cm}^3$ and $R_j$ is the jovian equatorial radius.

Maps of the jovian magnetic field with the corresponding polarization rate have been deduced according to the model described in Section 2.3. Results are shown in Fig. 6. One may note that the polarization rate for the dipolar field shows a regular and periodic pattern versus longitude. This rate may exceed a level of $2.83 \times 10^{-2}$ at the poles, a value that was derived for the maximum magnetic field strength so far assumed, thereby confirming the general property of enhanced polarization by the Hanle effect in the case of photons backscattering.

5.2.2. The $O_4$ based field model

In this model, the total magnetic field is defined as the sum of an internal magnetic field $\vec{B}_i$, the so-called $O_4$ component, and an external field $\vec{b}$ produced by a representative jovian magnetodisc that dominates the internal field at large distances (Connerney, 1981):

\begin{equation}
\vec{B} = \vec{B}_i + \vec{b},
\end{equation}

where $\vec{B}_i = -\vec{\nabla} V$ is expressed as the gradient of a scalar function $V$. With well-defined spherical coordinates, the flattening of Jupiter at its poles is taken into account as obtained by the Voyager radio observations (Lindal et al., 1981). A FORTRAN code provides the strength of the associated magnetic field (J. Connerney, p.c., 2004). From the

Fig. 6. Left: Magnetic field at the surface of Jupiter according to the dipole model. Because the field is symmetric, $B$ iso-contours are straight lines. Right: Polarization rate at the surface of Jupiter according to the dipole magnetic field.
field strength we derive the following results:

\[ B_x = B \sin \theta \cos \phi, \]  
\[ B_y = B \sin \theta \sin \phi, \]  
\[ B_z = B \cos \theta, \]  
\[ \theta_B = \frac{180}{\pi} \cos \left( \frac{B_z}{B} \right), \]  
\[ \chi_B = \frac{180}{\pi} \left( \acos \left( \frac{B_y}{B} \right) \right) \frac{B_y}{B_y}, \]

where \( B = B(r, \theta, \phi) \).

The polarization rates are thus calculated using the magnetic field direction parameters \( \theta_B \), \( \chi_B \) and the \( B \) strength as done in the previous section. Maps of the magnetic field and the corresponding polarization are shown in Fig. 7. It is interesting to note the wide regions at latitudes larger than \( \sim 30^\circ \) and longitudes between \( 140^\circ \) and \( 280^\circ \) for which the polarization average is higher than \( \sim 0.01 \). These regions show an average magnetic field strength that exceeds \( \sim 10 \) Gauss. One may note with interest that the polarization map precisely reflects the same topology of the magnetic field strength.

5.2.3. The \( O_6 \) based field model

This model uses the so-called \( O_6 \) internal model (Connorney, 1993) but with an external field defined as follows (Khurana, 1997):

\[ \vec{B}_e = \vec{\nabla} f \times \vec{\nabla} g, \]

where \( f \) and \( g \) are two scalar functions known as Euler potentials. Results obtained so far for the Khurana model (their FORTRAN code “case = 4” that fits four spacecraft data) are shown in Fig. 8 for both the magnetic field strength and the associated polarization rate. The magnetic field shows an extended polar zone of strength 13.45 Gauss with a corresponding polarization rate of 0.018. The same property of similitude between the field strength and the polarization rate maps are also confirmed for the \( O_6 \) model. The polarization picture is clarified in the following section when considering projected maps on the jovian disc—maps that will be of interest in the future to build observational strategy that would optimize the polarization/magnetic field detection.

5.3. Jupiter disc and off-disc projected polarization rate

As shown by Figs. 7 and 8, for both the \( O_4 \) and \( O_6 \) based models, the magnetic field strength and the polarization maps are similar, although the polarization rates for the \( O_6 \) model are slightly weaker. In the following we focus on the \( O_4 \) magnetic field model. The disc projected magnetic field strength and polarization rate maps calculated at CML = 200º are shown in Fig. 9. The planetary axis tilt has been properly included in the model calculations. The similarity between the two maps suggest that a visual inspection of a polarization map should immediately and quite accurately reveal the general topology of the magnetic field strength over the planetary disc. Consequently, a quick look at the projected maps clearly shows a spot in both the field strength and polarization rate with high contrasts against the equatorial regions. The spot is centered around longitude \( \sim -60^\circ \) from the selected CML \( 200^\circ \) with a width of \( \sim 60^\circ \), and corresponds to all latitudes above \( \sim 25^\circ \) north. To avoid any contamination from auroral regions, one should restrict the observations to latitudes smaller than 55º (Ballester et al., 1996; Clarke et al., 2002). Two major and interesting charac-

![Fig. 7. The \( O_4 \) based model. Left: Magnetic field strength at the surface of Jupiter. Right: Polarization rate corresponding to the magnetic field shown on the left.](image-url)
teristics of this particular region of the jovian disc are a strong polarization rate corresponding to a spot of high field strength and the high contrast with the other regions. However, the real jovian magnetic field at the ionospheric level may be different from the picture shown in Fig. 9. Nonetheless, from the previous calculations, a polarization mapping at two or more CML longitudes should reveal the gross topology of the jovian magnetic field. At minimum, any extended spot with peak field strength should immediately appear in the polarization maps. The Hanle effect thus provides a new technique for future detection of the jovian magnetic field strength at the planetary surface.

Additionally, a polarization mapping of the Lyα bulge region, an extended region centered on longitude ~90° System III, where a strong turbulence is expect in the outer layers of the jovian atmosphere (Ben Jaffel et al., 1993; Emerich et al., 1996), may appear disordered, revealing the random aspect of the corresponding ambient turbulent magnetic field (Ignace Richard, private communication). After decades of observations in the far and extreme UV, mapping of the Hanle effect polarization in the turbulent bulge region would then bring a new diagnostic that may unravel new properties of the exact origin of this long standing phenomena.

Fig. 8. The O₆ based model. Left: Map of the jovian surface magnetic field strength. Right: Polarization rate corresponding to the magnetic field shown on the left.

Fig. 9. Left: Magnetic field map on the apparent planetary disc at the jovian CML = 200°. The O₄ based model is used, taking into account the planetary axis 10° tilt. Right: Polarization rate on the apparent disc corresponding to the magnetic field shown on the left.
The next step is to check the polarimetry diagnostic off the disc of Jupiter, particularly in the coronal region of the planet and near the orbit of some of its satellites. For that purpose, we have plotted the polarization rate versus the radial distance from the planetary center expressed in units of jovian radius. We have selected a region of the disc such that the field strength is $\sim 10$ Gauss at the limb. As expected, the magnetic field strength drops sharply off-disc (see Fig. 10, left). The corresponding radial distribution of the polarization is shown in Fig. 10 (right panel). While a useful polarimetric diagnostic could be derived up to $2 R_J$ from the disc center, unfortunately, both the polarization rate and the Ly$\alpha$ brightness drop to a level that makes a detection quite difficult for distances corresponding to the orbit of Ganymede or Io. Using Fig. 10 (right) and according to recent HST/STIS and Galileo Ly$\alpha$ brightness distributions obtained off limb (Emerich et al., 2003; Gladstone et al., 2004), the proposed Hanle effect technique should be very promising for mapping the jovian magnetic field strength on the planetary disc up to the jovian corona (up to $\sim 1.2 R_J$). Moreover, simple opacity calculations using standard models of the jovian upper atmosphere show that the Ly$\alpha$ emission becomes optically thin above $\sim 1.04 R_J$, allowing to use unambiguously the full emission line in the Hanle effect diagnostic without any restriction on the spectral range.

6. Discussion of polarimetry techniques for the detection of the Ly$\alpha$ Hanle effect

Although the predicted polarization rates are high for localized regions on the jovian planetary disc, we need to clarify the instrumental options that may provide the most viable polarimetry diagnostic of the Hanle effect and give an accurate estimate of the magnetic field strength.

The three primary concerns in the design of an instrument for the study of the jovian Ly$\alpha$ Hanle effect are etendue (collecting area $\times$ field of view), throughput (effective area), and polarimetric sensitivity. These three combine somewhat competing needs to obtain the maximum number of photons while simultaneously probing the intensity of the signal along the two axes perpendicular and parallel to the scattering plane. A separate but equally important characteristic is that the instrument must be physically small enough to fit as a component instrument on a spacecraft. A Ly$\alpha$ polarimeter will have two components, a pre-filter that samples the polarization and scattering angle of the incoming light and a sensor system that detects and processes the signal. There are several options for each with different advantages. Ultimately the design of an instrument that is optimized for the study of the Hanle effect will come from the competing needs for spatial coverage, wavelength resolution, sensitivity, platform (i.e., at the focus of a large aperture telescope or mechanically collimated) and the vantage point (distance from Jupiter) from which the observations will be performed.

While use of the Hanle effect to measure small-scale magnetic fields in the Sun have become somewhat routine (Stenflo, 1994; Faurobert and Arnaud, 2003), achieving the necessary precision in FUV polarimetry presents significant technical challenges. Indeed, NUV–FUV polarimetry has only rarely been pursued for astronomical observations of any kind (e.g., ASTRO–WUPPE; Nordsieck et al., 1994, and D2A-Polar; Blamont et al., 1975), none of which have involved a planetary mission. The key difficulties lie with
Moreover, this configuration, with all other wavelengths having their wavefronts replaced by gratings. A SHS is tuned single “heterodyne” to the zero path location of the Michelson configuration in the ~30% range for peak polarizations of 10% or more. Recent designs in FUV polarimeters (Nordsieck and Harris, 1996) have used stressed LiF and MgF2 waveplates, combined with a diamond Brewster mirror to achieve significant improvement in throughput and polarimetric efficiency. Along with advances in optical coatings and detector quantum efficiencies, these improvements have opened new areas of astronomical study (e.g., Ignace et al., 1997).

Scientifically useful measurement of the jovian magnetic field from the Hanle effect requires precisions of ~1% for peak polarizations of ~2% (Fig. 9) or a s/n larger than 5000. Moreover, this s/n must be obtained at high spectral resolution in order to separate the multiply-scattered line core from the single-scattered wing, and must be accomplished in a compact format that can be included as a component instrument on a planetary mission. The relatively high (~10 kR) brightness of jovian dayglow provides a substantial photon flux; however, it is still essential to maximize the light gathering power, or etendue (FOV of field of view × collecting area), of any instrument used to observe it. Unfortunately, high resolution in grating spectrographs comes at the expense of FOV since the entrance aperture is reduced in size. The Echelle mode on HST/STIS is an excellent example. A 10-kR signal observed with the 0.2′′ × 0.2′′ aperture would yield a detected flux of significantly less than 1 event/s, leading to a s/n ~ 25 over an entire HST orbit, despite the advantage of a large 2.4-m input aperture.

Achieving the necessary etendue for high-resolution spectro-polarimetry requires moving away from grating spectrograph designs. Fortunately, there is an instrumental approach that simultaneously maximizes etendue and spectral resolution: interferometry. While interferometers take many forms, the basic implementation naturally maximizes etendue because, primarily, the size of the entrance aperture of the instrument does not affect the spectral resolution of the output power spectrum as strongly as it does for grating spectrographs. This has two significant advantages for narrow band spectroscopy of extended emission features like the jovian dayglow. First, the large acceptance angle of an interferometer maximizes the number of photons from the target that reach the instrument, and second, it permits the use of the instrument in either a mechanically collimated format or at the focus of a very modest telescope. Of the various interferometer designs, the one with the most heritage in space-based UV astronomy is the spatial heterodyne spectrograph (SHS). The basic design of an SHS is similar to the more familiar Michelson interferometer, with the mirrors replaced by gratings. A SHS is tuned single “heterodyne” wavelength at the zero path location of the Michelson configuration, with all other wavelengths having their wavefronts rotated symmetrically away from parallel by the dispersing action of the gratings. At the output, the wavefronts interfere, thereby forming a two-dimensional fringe pattern with a frequency directly tied to the wavelength separation from heterodyne reference wavelength. A Fourier transform of the interference pattern recovers the spectrum. Since neither the aperture of the instrument nor the format of the detector affect the spectral resolution of the instrument, the full resolving power of the illuminated grating is obtained. Indeed, a challenge for SHS is to keep the resolution manageable low. For additional details on SHS designs, the reader is referred to the work of Harlander et al. (1992), Roesler and Harlander (1990), and Harris et al. (2003). A simple calculation based on conservative estimates of optical element efficiency suggests that an instrument could be used at the focus of a small (20 cm) diameter telescope to observe the jovian dayglow from the vantage point of Europa, achieving a s/n ~ 5000 in a 1.5–2 h integration time with a spatial resolution on Jupiter of ~1000 km or less. Such an instrument could achieve the 1% accuracy on the polarization rate required by our model calculations to obtain at least 10% accuracy on the strength of the jovian surface magnetic field.

The primary disadvantage of the SHS technique is multiplexing noise in the output. Because the spectrum is obtained from a Fourier transform of the full interference pattern, all of the photons that are counted contribute to the noise in every spectral element. This reduces the efficiency of the technique significantly for broad continuum sources, but is less of a problem in the evaluation of isolated lines. Multiplexing introduces a secondary requirement that the optical system be designed to limit off axis and out of band light, which, while unresolved in the interference pattern, will nevertheless contribute to the overall noise. The off axis contribution can be minimized with baffling, and it should be noted that its presence has no effect on spectral resolution, nor does it contribute to the recovered spectrum in anyway aside from adding to statistical noise. Bandpass filtering, analogous to an order separation filter in an echelle spectrograph should also be employed to minimize noise producing unresolved signal from outside the tuned spectral range. Other sources of uncertainty, such as the extended wing of the multiply scattered jovian dayglow line core or background (e.g., interplanetary H, and/or geocorona) are largely mitigated by the high resolution of the instrument and can be accounted for in post processing analysis of the observed polarization. In any case, for jovian dayglow, the single scattered wing of the broad feature contribute an amount of emission equivalent to that in the line-core, rendering the low-level tail of the line-center negligible to the target emissions past few characteristics widths. Indeed, the multiplex noise from line center emission will have a more significant effect on s/n than the extended line-shape, provided the resolution is high enough.

UV SHS instruments have been built and flown to study emissions in the NUV (at 3000 Å; Harlander et al., 2002)
and FUV (at 1541 Å; Watchorn et al., 2001, and 1215 Å; Stephan et al., 2001). For the near future, a NASA/CNES suborbital project, INSPIRE, seeks to design and fly a UV optimized polarimetric SHS at Lyα with a first attempt in ∼2007 to detect the Hanle effect from Jupiter Lyα and estimate the averaged strength of the jovian surface magnetic field (see http://www.iap.fr/inspire for more details). INSPIRE is indeed a prototype mission envisioned to validate the Lyα polarimeter-SHS in space in an effort to prepare future planetary missions like JIMO. A sub-orbital measurement is limited by its relatively short observing time ∼300 s and a contaminating background from the geocorona. However, our preliminary models indicate that a well designed instrument that is properly etendue-matched to the angular size of Jupiter and launched at the lowest ebb of geocoronal emission (2.5 kR) will be limited primarily the photon statistics of dayglow and can achieve better than 1% polarimetric sensitivity over the full disc of the planet.

One alternative option with SHS that could achieve much higher-etendue is the so-called “field-widened” implementation (Harlander et al., 1992) that places transmitting wedges in front of the gratings to reduce second order effects that reduce contrast in the output interference pattern. The field-widened design, which has been used in spacecraft applications (Watchorn et al., 2001; Harlander et al., 2002), increases the maximum acceptance angle by a factor of ∼10 over traditional SHS designs. The impact of this on instrument etendue is enormous: compared with the HST-STIS mode described above, a field widened SHS designed to observe H Lyα at R = 100,000 would have more than 5000 times greater etendue in a format that would easily fit in a planetary mission. Field-widening has not been implemented at 1215 Å; however, it is conceptually similar to designs already built for use in the FUV (1541 Å; Watchorn et al., 2001). At 1215 Å however, the transmission of the beam splitter and wedges will affect the efficiency of the instrument and the wider field of view will require larger input optics to correctly match with the desired target spatial resolution. Nevertheless this option could significantly improve prospects for reaching the required s/n.

7. Conclusions

In this paper, we evaluated the impact of the Hanle effect on the polarization of the jovian Lyα emission line and its application to measuring the strength of the jovian magnetic field at the planetary surface and in different regions of the magnetosphere. To investigate this new technique in Planetary Sciences, we made use of the formalism developed in Solar Physics (see, for instance, Bommier, 1997a, and references therein), which provides a theoretical background for deriving the dependency of the polarization rate on the local magnetic field. To test the sensitivity of the jovian Lyα emission polarization to the ambient magnetic field, we considered three models: a dipole field model for reference purposes, as well as two other models defined as the sum of internal and external magnetic field components derived from in situ measurements made in the jovian magnetosphere by several spacecraft. We derived the corresponding polarization rates and visualized them as 2D iso-contours over the planetary sphere.

The main result of our work is that with the photon backscattering process assumed for the formation of jovian Lyα dayglow emission, the Hanle effect enhances the polarization rate, contrary to the case of the solar corona (scattering at right angle) where a depolarizing effect is obtained. Polarization rates exceeding 2% were obtained in some regions of the planetary disc, making a detection quite possible either remotely or from an orbiter around Jupiter. We have also found a similarity between the field strength and polarization maps—a similarity suggesting that visual inspection of a polarization map should immediately and quite accurately reveal the topology of the magnetic field strength over the planetary disc. For all realistic magnetic field models considered, a zone of enhanced polarization is located at latitudes higher than 25° and longitudes centered on 200° (System III). Off-disc, we showed that the Hanle technique should be useful for mapping the magnetic field in the jovian corona. At the jovian satellites orbit, the Hanle effect polarimetric diagnostic will be difficult to measure because both the Lyα backscattered emission and the corresponding polarization are rather weak. Similar model calculations for Saturn predict a polarization rate that should not exceed $p = 3 \times 10^{-5}$, a rather weak value to be detected remotely from Earth. Finally, instrumental options that may optimize the detection of the Hanle effect with a 1% accuracy (at least 10% on the magnetic field strength) are suggested.

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