Cosmological parameters from second- and third-order cosmic shear statistics

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Outline of the talk

1. Motivation
2. Cosmic shear
3. Optimizing cosmological parameter estimation
   3.1 Survey design
   3.2 Combining second and third-order shear statistics
4. Outlook
1. Motivation
In the last decade: most observations consistent with flat \( \Lambda \text{CDM} \) “concordance model”. Parameters known to \( \sim 5\text{-}10\% \) accuracy.

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<tbody>
<tr>
<td>dark energy</td>
<td>( \Omega = 0.72 )</td>
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<tr>
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**Cosmology**

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Seljak et al. 2005

Cosmological parameters from second- and third-order cosmic shear statistics – p.4/36
Cosmology

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Cosmological parameters from second- and third-order cosmic shear statistics – p.4/36
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[Seljak et al. 2005]

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[Seljak et al. 2005]
Large-scale structure

We want to understand:

- What were the seeds in the early Universe out of which today’s structure evolved? [inflation]
- How did structure form?
- What is the amplitude of the density fluctuations?

[Springel et al. 2005]
Cosmological parameters from second- and third-order cosmic shear statistics – p.5/36
2. Cosmic shear
Cosmic shear = coherent distortion of images of distant galaxies caused by matter inhomogeneities on large scales.

Weak gravitational lensing regime, statistical detection.

Cumulative distortion along the line of sight $\rightarrow$ sensitive to projected matter distribution $\kappa$.

Depends on source galaxy distribution $p(\tilde{z})dz$. 

Cosmological parameters from second- and third-order cosmic shear statistics – p.7/36
Cosmic shear

Distortions lead to observable mutual alignment or correlation of orientation of background galaxy images.

Amplitude of correlation depends on “lumpiness” of LSS.
Cosmic shear

- Sensitive to luminous & dark matter
  \[\Rightarrow\] measures density fluctuations without bias

\[\sigma_8 = \text{rms of matter fluctuations in spheres of } 8\ h^{-1} \text{ Mpc radius}\]

- Complementary to e.g. CMB anisotropy experiments

[Contaldi et al. 2003]
Cosmic Shear

- Probes non-Gaussianity of the LSS (using higher-order statistics) due to non-linear evolution, gravitational collapse
Cosmic Shear

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Aperture Mass Statistics

Mass overdensity $\kappa$

$$M_{\text{ap}}(\theta) = \int d^2x \ U_\theta(x) \kappa(\vec{x})$$

$$= \int d^2x \ Q_\theta(x) \gamma_t(\vec{x})$$

Tangential shear in aperture $\gamma_t$

Cosmological parameters from second- and third-order cosmic shear statistics – p.11/36
convergence field $\kappa$

shear field $\gamma$

Cosmological parameters from second- and third-order cosmic shear statistics – p.12/36
\[ M_{\text{ap}}(\theta) = \int d^2x \, U(\theta) \kappa(\vec{x}') \]

\( M_{\text{ap}}(\theta) \) is a smoothed version of the projected density \( \kappa \).

The variance \( \langle M_{\text{ap}}^2(\theta) \rangle \) is a measure of the amount of fluctuations in \( \kappa \) on scales \( \sim \theta \).

\[ \leftrightarrow \text{“lumpiness” of LSS} \]
\[ \leftrightarrow \text{power spectrum of } \kappa. \]

\( (P_\kappa \text{ projection of 3D power spectrum}) \)

Cosmological parameters from second- and third-order cosmic shear statistics – p.13/36
convergence field $\kappa$

shear field $\gamma$
3. Optimizing cosmological parameter estimation

3.1 Survey design
Cosmic shear surveys

- large area (∼ 1...1000 square degree)
- empty fields (random pointings)
- large range of angular scales (1...100 arc minutes)
- observe different regions on sky to reduce “cosmic variance”

Ongoing and future multi-color surveys:

<table>
<thead>
<tr>
<th>survey</th>
<th>area [sq deg]</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTIO</td>
<td>75</td>
<td>1996–2000</td>
</tr>
<tr>
<td>VIRMOS-DESCART</td>
<td>12</td>
<td>1999–2001</td>
</tr>
<tr>
<td>CFHTLS</td>
<td>170</td>
<td>2003–</td>
</tr>
<tr>
<td>KIDS</td>
<td>1500</td>
<td>2006–2010?</td>
</tr>
<tr>
<td>LSST</td>
<td>all sky</td>
<td>2012–</td>
</tr>
<tr>
<td>DUNE</td>
<td>20 000</td>
<td>2011–</td>
</tr>
<tr>
<td>SNAP/JDEM</td>
<td>300-1000</td>
<td>?</td>
</tr>
</tbody>
</table>
Survey strategies

small cosmic variance vs. large angular scales

total: 300 images \(\times 13' \times 13' = 14 \, \text{deg}^2\)
Q: What is best survey strategy to constrain cosm. parameters?

The more accurate we can measure the shear correlation and the broader the range of scales, the better we will be able to determine cosmological parameters.

Measurement errors encoded in covariance matrix.

Estimator $x_i$, e.g. $x_i = \langle M_{ap}^2(\theta_i) \rangle$.

Covariance matrix $\text{Cov}_{ij} = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$.
Covariance includes errors due to:

- intrinsic shapes of galaxies (not circular)
- discrete number of galaxies
- cosmic variance

Depends on geometrical setting of the survey!
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Not included:

- Systematic errors
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- Systematic errors
- Non-Gaussian features of shear field
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Not included:

- Systematic errors
- Non-Gaussian features of shear field

Analytical expressions (Gaussian case)

[Schneider, van Waerbeke, MK & Mellier 2002]

Integration of these expressions using Monte-Carlo like method: time-consuming, involves four-point statistics [MK & Schneider 2004]
Measurement errors \implies errors on parameters:

Covariance \implies \text{Fisher information matrix } F_{\alpha\beta}

F_{\alpha\beta} \implies \text{lower bound on } 1\sigma \text{ error for parameters:}

\textbf{minimum variance bound}

\begin{align*}
\sigma(p_{\alpha}) & \leq \frac{1}{\sqrt{F_{\alpha\alpha}}} \\
\text{one parameter only} & \\
\sigma(p_{\alpha}) & \leq \sqrt{(F^{-1})_{\alpha\alpha}} \\
\text{marginalized over several parameters} & 
\end{align*}
Comparing surveys

1σ errors from $\langle M_{ap}^2 \rangle$

300/N patches, each with N pointings

Uncorrelated lines of sight:

<table>
<thead>
<tr>
<th>N</th>
<th>$\sigma(\Omega_m)$</th>
<th>$\sigma(\Gamma)$</th>
<th>$\sigma(\sigma_8)$</th>
<th>$\sigma(n_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.53</td>
<td>0.16</td>
<td>0.77</td>
<td>0.20</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
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Karhunen-Loève technique

[Teegmark, Taylor & Heavens 1997]

Linear data compression $\tilde{M}_i = \sum_j T_{ij} \langle M_{ap}^2(\theta_j) \rangle$ with as little information loss as possible.

Consider one parameter $p_\alpha$. KL data compression leaves mvb

$\sigma(p_\alpha) = 1/\sqrt{F_{\alpha\alpha}}$ invariant!
Original data vector is Fourier transform of $\kappa$ power spectrum

$$\langle M_{ap}^2(\theta) \rangle = \frac{1}{2\pi} \int_0^\infty d\ell \, \ell \, P_\kappa(\ell) \left( \frac{24J_4(\ell\theta)}{(\ell\theta)^2} \right)^2$$

Compressed data vector will sample power spectrum differently

$$\tilde{M}_i = \frac{1}{2\pi} \int_0^\infty d\ell \, \ell \, P_\kappa(\ell) W_i(\ell)$$

“window function”
KL window functions
(\(\Omega_m, \sigma_8\)), (\(\Gamma, n_s\)) near-degenerate parameters.
Karhunen-Loève + SVD

[Tegmark, Taylor & Heavens 1997]

Linear data compression \( \tilde{M}_i = \sum_j T_{ij} \langle M^2_{ap}(\theta_j) \rangle \) with as little information loss as possible. Consider one parameter \( p_\alpha \). KL data compression leaves \( \sigma(p_\alpha) = 1/\sqrt{F_{\alpha\alpha}} \) invariant!
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All parameters simultaneously: combine compression matrices \( T \) from individual optimization into big matrix.

Get rid of redundancies: Singular Value Decomposition
KL + SVD Window functions

Only two modes are needed to encompass cosmological information. [MK & Munshi 2005]
Only two modes are needed to encompass cosmological information. [MK & Munshi 2005]

Cosmological parameters from second- and third-order cosmic shear statistics – p.26/36
Conclusions (survey strategy)

- Shear information on medium and large scales crucial.
- 25% improvement on the (minimal) 1σ errors possible.
- KL technique shows which scales are sampled in an optimal way.
- Two “modes” sufficient to constrain cosmological parameters from cosmic shear (without redshift information).
- Two “degeneracy groups” of parameters: \((\Omega_m, \sigma_8)\) and \((\Gamma, n_s)\).
3. Optimizing cosmological parameter estimation

3.2 Combining second- and third-order statistics
Higher-order statistics

Second-order: \( \langle M_{ap}^2(\theta) \rangle \)

[Schneider, MK & Lombardi 2004]: Generalized third-order aperture mass statistics \( M_{ap}^3(\theta) \) contains much more information about cosmology (cross-correlation of LSS on different scales).
Higher-order statistics

Second-order: \( \langle M^2_{\text{ap}}(\theta) \rangle \)

Third-order: \( \langle M^3_{\text{ap}}(\theta) \rangle \)
Higher-order statistics

Second-order: $\langle M^2_{ap}(\theta) \rangle$

Third-order: $\langle M^3_{ap}(\theta) \rangle$

[Schneider, MK & Lombardi 2004]: Generalized third-order aperture mass statistics $\langle M^3_{ap}(\theta_1, \theta_2, \theta_3) \rangle$ contains much more information about cosmology (cross-correlation of LSS on different scales).
convergence $\kappa$

shear $\gamma$

$M_{ap}^2(\theta)$

$M_{ap}^3(\theta_1, \theta_2, \theta_3)$
Second- and third-order statistics (slightly) different dependence on cosmological parameters.

→ Combine both to get better constraints.

Covariance matrices from ray-tracing simulations

[thanks to T. Hamana!]

Fisher matrix to obtain $1\sigma$ errors on cosm. parameters.

[MK & Schneider 2005]
Cosmological Parameters

$\Omega_m$  dark matter density

$\sigma_8$  power spectrum norm.

$\Gamma$  shape parameter

$\langle M^2_{ap} \rangle$
Cosmological parameters from second- and third-order cosmic shear statistics – p.32/36
Cosmological Parameters

\begin{align*}
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Cosmological parameters from second- and third-order cosmic shear statistics – p.32/36

\[ \Omega_m \] dark matter density
\[ \sigma_8 \] power spectrum norm.
\[ \Gamma \] shape parameter
Cosmological Parameters

- $\Gamma$ shape parameter
- $n_s$ initial spectral index
- $\Omega_\Lambda$ cosm. constant
- $z_0$ source redshift

Cosmological parameters from second- and third-order cosmic shear statistics – p.33/36
Conclusions ($2^{nd} + 3^{rd}$ order)

- Combination of $\langle M_{ap}^2 \rangle$ and (generalized) $\langle M_{ap}^3 \rangle$ can partially lift $\Omega_m-\sigma_8$ degeneracy.
- $\Gamma$ and $n_s$ stay near-degenerate.
- Improvement on cosmological parameters by $\sim$ factor 10 ($1\sigma$ error).
5. Outlook

Ongoing and future cosmic shear surveys . . .

- are most promising to constrain dark energy
- will measure third-order statistics with high significance
- will have smaller systematic errors than theoretical uncertainties.

Therefore, we need . . .

- realistic modelling of dark energy
- strategy for constraining dark energy with cosmic shear
- efficient methods to extract higher-order statistics from data
- to improve models of LSS ($N$-body, ray-tracing simulations).
References