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Gravitational-wave inspiral of compact binary systems to 7/2 post-Newtonian order

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The inspiral of compact binaries, driven by gravitational-radiation reaction, is investigated through 7/2 post-Newtonian (3.5PN) order beyond the quadrupole radiation. We outline the derivation of the 3.5PN-accurate binary's center-of-mass energy and emitted gravitational flux. The analysis consistently includes the relativistic effects in the binary's equations of motion and multipole moments, as well as the contributions of tails, and tails of tails, in the wave zone. However, the result is not fully determined because of some physical incompleteness, present at the 3PN order, of the model of point particle and the associated Hadamard-type self-field regularization. The orbital phase, whose prior knowledge is crucial for searching and analyzing the inspiral signal, is computed from the standard energy balance argument.

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A few years ago it was recognized [1] that improved waveform modeling is crucial to construct templates for searching and measuring gravitational waves from inspiraling compact binaries with laser-interferometric detectors such as the Laser Interferometric Gravitational Wave Observatory (LIGO) and VIRGO. Since a large number of orbital cycles are observable in the frequency band of the detectors, the measurement, using the technique of matched filtering, will be extremely sensitive to those parameters that affect the inspiral rate and thus the orbital phase evolution. The orbital phase (which is driven by gravitational-radiation reaction) is therefore the crucial quantity to be monitored for these experiments. Measurement-accuracy analyses [2] have shown that a very high post-Newtonian (PN) prediction, probably the third post-Newtonian, or even the 3.5PN one (i.e., $1/c^7$), in the case of neutron-star binaries, would be required. Only then would the templates be accurate enough over most of the inspiral phase, with reduced cumulative phase lags, so that the phasing errors are not significant when one attempts to extract the values of the binary's parameters (essentially the masses and spins) from the data. Having in hand such high-order post-Newtonian expressions, one could apply resummation methods to further improve the convergence of the post-Newtonian series and make it even more accurate for searches as well as parameter estimations [3,4]. In this Rapid Communication, to provide the essential theoretical input for gravitational-wave data analysis [1–4], we compute the orbital phase of compact binaries, in both the time and frequency domains, in the adiabatic approximation, at 3.5PN order. Numerical relativity or approaches such as [4] could describe the plunge and merger phases. The latter approach starts from the post-Newtonian expansion and goes beyond the adiabatic approximation. Appropriate to the inspiral regime [1], we treat the compact bodies as structureless, nonspinning point particles, moving on quasicircular orbits. Spin effects are known up to 2.5PN order [5] and may be added if necessary.

The first ingredient in the theoretical analysis is the equation of motion of the binary, which is used primarily for the calculation of the center-of-mass energy E that is conserved in the absence of gravitational-radiation reaction. Recently, the equation of motion of compact binaries at 3PN order has been obtained by means of two different methods, with equivalent results. Jaranowski and Schäfer [6] and Damour, Jaranowski, and Schäfer [7,8] employ the Arnowitt-Deser-Misner (ADM) Hamiltonian formalism of general relativity, while Blanchet and Faye [9,10] and de Andrade, Blanchet, and Faye [11] proceed with the post-Newtonian iteration of the Einstein field equations in harmonic coordinates. Since the binary's orbit would have been circularized by the radiation reaction, the equation of motion is of the form

$$dv^i/dt = -\omega^2 x^i + c^{-5} F^i_{\mathrm{reac}}, \tag{1}$$

where $x^i = y_1^i - y_2^i$ is the vector separation between the two particles, $v^i = dx^i/dt$ the relative velocity, and ω the orbital angular frequency ($\omega = 2\pi/P$, where P is the period). We denote by F^i_{reac} the standard radiation reaction—a resistive force opposite to the relative velocity, which arises dominantly at 2.5PN order. Through 3PN order, the orbital frequency is related to the distance $r = |\mathbf{x}|$ in harmonic coordinates (via the post-Newtonian parameter $\gamma = Gm/rc^2$) by [9]

$$\begin{aligned}\omega^2 = \frac{Gm}{r^3}\Big(&1 + (-3+\nu)\gamma + \left(6 + \tfrac{41}{4}\nu + \nu^2\right)\gamma^2 \\ &+ \{-10 + [-\tfrac{67759}{840} + \tfrac{41}{64}\pi^2 + 22\ln(r/r_0') \\ &+ \tfrac{44}{3}\lambda]\nu + \tfrac{19}{2}\nu^2 + \nu^3\}\gamma^3\Big).\end{aligned} \tag{2}$$

Mass parameters are the total mass $m = m_1 + m_2$ and the symmetric mass ratio $\nu = m_1 m_2/m^2$ satisfying $0 < \nu \leqslant 1/4$ (the reduced mass is then $\mu = m\nu$). The 3PN coefficient depends on two arbitrary constants: a length scale r_0' entering the logarithm and the constant λ. It was shown in Ref. [9] that r_0' is merely linked with the choice of harmonic coordinates and has therefore no physical meaning, as it can be eliminated by a change of gauge. By contrast, λ represents a physical indeterminacy, in the form of a purely numerical constant (e.g., a rational fraction), and is probably associated with an incompleteness of the Hadamard-type method for regularizing the infinite self-field of point particles [10], which is used to cope with the model of compact objects idealized by Dirac functions (for general, noncircular orbits, it is impossible to reabsorb λ into a redefinition of the gauge constant r'_0). The presence of λ may be associated with the fact that many integrals composing the equation of motion, when taken *individually*, start depending, from 3PN order, on the internal structure of the bodies, even in the limit where their size tends to zero. However, when considering the *full* equation of motion, we finally expect λ to be independent of the internal structure of the compact bodies. The constant λ is equivalent to the static ambiguity parameter ω_{static} introduced in Refs. [6], [7], in the sense that $\lambda=-\frac{3}{11}\omega_{\text{static}}-1987/3080$. Recently, the value $\omega_{\text{static}}=0$ has been obtained by means of a dimensional regularization supplementing the ADM-Hamiltonian formalism [8]. This result would mean that $\lambda=-1987/3080$ (but we keep λ unspecified in this discussion).

From now on we shall use in place of the angular frequency ω the dimensionless variable $x=(\pi Gmf/c^3)^{2/3}$, where $f=2/P=\omega/\pi$ is the frequency of the gravitational-wave signal at the dominant harmonic. By inverting Eq. (2) one finds γ in terms of the variable x, which we shall now consider as an alternative ordering post-Newtonian parameter,

$$\gamma=x\Big(1+\big(1-\tfrac{1}{3}\nu\big)x+\big(1-\tfrac{65}{12}\nu\big)x^2+\Big\{1+\Big[-\tfrac{10151}{2520}-\tfrac{41}{192}\pi^2-\tfrac{22}{3}\ln(r/r'_0)-\tfrac{44}{9}\lambda\Big]\nu+\tfrac{229}{36}\nu^2+\tfrac{1}{81}\nu^3\Big\}x^3\Big). \tag{3}$$

As the 3PN equation of motion for general orbits derives from a Lagrangian [11] (neglecting the radiation reaction), one can straightforwardly compute the associated 3PN conserved energy. The result, when specialized to circular orbits, reads

$$E=-\tfrac{1}{2}\mu c^2\gamma\Big(1+\big(-\tfrac{7}{4}+\tfrac{1}{4}\nu\big)\gamma+\big(-\tfrac{7}{8}+\tfrac{49}{8}\nu+\tfrac{1}{8}\nu^2\big)\gamma^2+\Big\{-\tfrac{235}{64}+\Big[\tfrac{106301}{6720}-\tfrac{123}{64}\pi^2+\tfrac{22}{3}\ln(r/r'_0)-\tfrac{22}{3}\lambda\Big]\nu+\tfrac{27}{32}\nu^2+\tfrac{5}{64}\nu^3\Big\}\gamma^3\Big). \tag{4}$$

The good thing to do next is to reexpress this energy in terms of the post-Newtonian parameter x. Indeed, as x is directly related to the orbital period, the energy will be form invariant (the same in different coordinate systems). We find [7,9]

$$E=-\tfrac{1}{2}\mu c^2x\Big\{1+\big(-\tfrac{3}{4}-\tfrac{1}{12}\nu\big)x+\big(-\tfrac{27}{8}+\tfrac{19}{8}\nu-\tfrac{1}{24}\nu^2\big)x^2+\Big(-\tfrac{675}{64}+\Big[\tfrac{209323}{4032}-\tfrac{205}{96}\pi^2-\tfrac{110}{9}\lambda\Big]\nu-\tfrac{155}{96}\nu^2-\tfrac{35}{5184}\nu^3\Big)x^3\Big\}. \tag{5}$$

As expected, the latter expression is free of the unphysical gauge constant r'_0. Since it can be checked that for circular orbits there are no terms of order $x^{7/2}$, the energy (5) is in fact valid up to 3.5PN order. In the test-mass limit $\nu\rightarrow 0$, we recover the energy of a particle with mass μ in a Schwarzschild background of mass m, i.e., $E_{\text{test}}=\mu c^2[(1-2x)(1-3x)^{-1/2}-1]$, when developed to 3.5 PN order.

The second ingredient in this analysis concerns the gravitational waveform generated by the compact binary. More precisely, we need to compute the binary's total energy flux at infinity, or gravitational luminosity $\mathcal{L}$, in the post-Newtonian approximation. This calculation should take into account the relativistic corrections linked with the description of the source (multipole moments), as well as the nonlinear effects in the propagation of the waves from the source to the far zone. We have applied here a particular wave-generation formalism [12–14], valid for slowly moving sources, in which the exterior field is parametrized by some specific multipole moments, formally valid to any post-Newtonian order [14], and where the observables at infinity are connected to the source moments by some nonlinear (post-Minkowskian) functional relations, taking into account the various effects of tails (see, e.g., [13]). The formalism has already been specialized to the case of inspiral waveforms at the 2.5 PN level by Blanchet, Damour, and Iyer [15]. Furthermore, a different formalism, devised by Will and Wiseman [16], was independently applied to this problem and reached equivalent results, reported jointly in Ref. [17], at 2PN order. The crucial input of any post-Newtonian computation of the flux is the mass quadrupole moment (indeed the required post-Newtonian precision of the higher moments is smaller). The 3PN quadrupole moment for circular binary orbits, say, $I_{ij}=\mu\,[A\hat{x}_{ij}+B(r^2/c^2)\hat{v}_{ij}]$, where we neglect a 2.5PN term and denote, e.g., $\hat{x}_{ij}=x_ix_j-\frac{1}{3}\delta_{ij}r^2$, has been obtained recently by Blanchet, Iyer, and Joguet [18], who find the result

$$A=1+\big(-\tfrac{1}{42}-\tfrac{13}{14}\nu\big)\gamma+\big(-\tfrac{461}{1512}-\tfrac{18395}{1512}\nu-\tfrac{241}{1512}\nu^2\big)\gamma^2+\Big\{\tfrac{395899}{13200}-\tfrac{428}{105}\ln(r/r_0)+\Big[\tfrac{139675}{33264}-\tfrac{44}{3}\ln(r/r'_0)-\tfrac{44}{3}\xi-\tfrac{88}{3}\kappa\Big]\nu+\tfrac{162539}{16632}\nu^2+\tfrac{2351}{33264}\nu^3\Big\}\gamma^3, \tag{6a}$$

$$B=\tfrac{11}{21}-\tfrac{11}{7}\nu+\big(\tfrac{1607}{378}-\tfrac{1681}{378}\nu+\tfrac{229}{378}\nu^2\big)\gamma+\Big\{-\tfrac{357761}{19800}+\tfrac{428}{105}\ln(r/r_0)+\Big[-\tfrac{75091}{5544}+\tfrac{44}{3}\zeta\Big]\nu+\tfrac{35759}{924}\nu^2+\tfrac{457}{5544}\nu^3\Big\}\gamma^2. \tag{6b}$$

Note the two types of logarithms entering these formulas at 3PN order. One type involves the same scale r'_0 as in the equation of motion [see Eqs. (2)–(4)]; the other one contains a different length scale r_0, which is exactly the constant present in the general formalism of Refs. [12–14]. As we know that the constant r'_0 is pure gauge, it will disappear from our physical results at the end. As for r_0, it merely represents a convenient scale entering the definition of the source multipole moments in Ref. [14] and should cancel out when considering the complete multipole expansion of the field exterior to the source. On the other hand, besides the harmless constants r'_0 and r_0, there are three unknown dimensionless parameters in Eqs. (6): ξ, κ, and ζ. These parameters are analogous to the constant λ in the equations of motion (see Ref. [18] for their definition in the general case of noncircular orbits). They probably reflect an incompleteness of the standard Hadamard self-field regularization used in [18]. It is possible that the more sophisticated regularization proposed in Ref. [10] could determine some (but maybe not all) of these parameters. However, we shall see that, in the case of circular orbits, the energy flux depends only on one combination of them, $\theta=\xi+2\kappa+\zeta$, and furthermore that this constant θ enters the energy flux at exactly the same level as λ, so that the luminosity given by Eq. (9) below depends on one and only one combination of θ and λ [to compute the flux one needs the time derivatives of the moment (6), and λ comes from replacing the accelerations by the equations of motion (1), (2)]. More work should be done to determine the values of θ and λ.

Through 3.5PN order, the result concerning the ''instantaneous'' part of the total energy flux, i.e., that part which is generated solely by the multipole moments of the source (not counting the tails), is [18]

$$\mathcal{L}_{\rm inst}=(32c^5/5G)\,\nu^2\gamma^5\Big(1+\big(-\tfrac{2927}{336}-\tfrac{5}{4}\nu\big)\gamma+\big(\tfrac{293383}{9072}+\tfrac{380}{9}\nu\big)\gamma^2+\Big\{\tfrac{53712289}{1108800}-\tfrac{1712}{105}\ln(r/r_0)$$
$$+\Big[-\tfrac{332051}{720}+\tfrac{123}{64}\pi^2+\tfrac{110}{3}\ln(r/r'_0)+44\lambda-\tfrac{88}{3}\theta\Big]\nu-\tfrac{383}{9}\nu^2\Big\}\gamma^3\Big), \tag{7}$$

where $\theta=\xi+2\kappa+\zeta$. The first term represents the Newtonian energy flux coming from the usual quadrupole formalism. To the latter instantaneous part of the flux, we must add the nonlinear tail effects in the wave zone, which have already been calculated to 3.5PN order in Ref. [13] [see Eqs. (5.5a) and (5.9) there]. We find

$$\mathcal{L}_{\rm tail}=(32c^5/5G)\,\nu^2\gamma^5\Big\{4\pi\gamma^{3/2}+\big(-\tfrac{25663}{672}-\tfrac{109}{8}\nu\big)\pi\gamma^{5/2}+\Big[-\tfrac{116761}{3675}+\tfrac{16}{3}\pi^2-\tfrac{1712}{105}C-\tfrac{856}{105}\ln(16\gamma)+\tfrac{1712}{105}\ln(r/r_0)\Big]\gamma^3$$
$$+\big(\tfrac{90205}{576}+\tfrac{3772673}{12096}\nu+\tfrac{32147}{3024}\nu^2\big)\pi\gamma^{7/2}\Big\}, \tag{8}$$

where $C=0.577...$ denotes the Euler constant. What we call here $\mathcal{L}_{\rm tail}$ is in fact a complicated sum of ''tails,'' ''tail squares,'' and ''tails of tails,'' as determined in Ref. [13]. It is quite remarquable that so small an effect as a ''tail of tail,'' which constitutes the whole 3PN coefficient in Eq. (8), should be relevant to the present computation, which is aimed at preparing the ground for a forthcoming experiment. As we can see, the constant r_0 drops out from the sum of the instantaneous (7) and tail (8) contributions—which is normal and constitutes a first test of the calculation. However, the gauge constant r'_0 does not seem to disappear at this stage, but that is simply due to our use in Eqs. (7), (8) of the post-Newtonian parameter γ, which depends via the equation of motion on the choice of harmonic coordinates. After substituting the frequency-related parameter x in place of γ; i.e., making consistent use of the relation (3), we find that r'_0 does cancel as well—which represents another test, showing the consistency between the two computations, in harmonic coordinates, of the equation of motion and multipole moments. Finally we obtain

$$\mathcal{L}=(32c^5/5G)\,\nu^2x^5\Big\{1+\big(-\tfrac{1247}{336}-\tfrac{35}{12}\nu\big)x+4\pi x^{3/2}+\big(-\tfrac{44711}{9072}+\tfrac{9271}{504}\nu+\tfrac{65}{18}\nu^2\big)x^2+\big(-\tfrac{8191}{672}-\tfrac{535}{24}\nu\big)\pi x^{5/2}+\Big(\tfrac{6643739519}{69854400}+\tfrac{16}{3}\pi^2$$
$$-\tfrac{1712}{105}C-\tfrac{856}{105}\ln(16x)+\Big[-\tfrac{11497453}{272160}+\tfrac{41}{48}\pi^2+\tfrac{176}{9}\lambda-\tfrac{88}{3}\theta\Big]\nu-\tfrac{94403}{3024}\nu^2-\tfrac{775}{324}\nu^3\Big)x^3+\big(-\tfrac{16285}{504}+\tfrac{176419}{1512}\nu+\tfrac{19897}{378}\nu^2\big)\pi x^{7/2}\Big\}. \tag{9}$$

The last test (but not the least) is that Eq. (9) is in perfect agreement, in the test-mass limit $\nu\to 0$ for one of the bodies, with the result following from linear black-hole perturbations obtained by Ref. [19] (see also Refs. [20]). In particular, the rational fraction 6643739519/69854400, which is a sum of other fractions appearing in both Eqs. (7) and (8), comes out exactly the same as in the black-hole perturbation theory [19].

TABLE I. Contributions to the accumulated number $\mathcal{N}=(1/\pi)(\phi_{\rm ISCO}-\phi_{\rm seismic})$ of gravitational-wave cycles. Frequency entering the bandwidth is $f_{\rm seismic}=10\,{\rm Hz}$; terminal frequency is assumed to be at the Schwarzschild innermost stable circular orbit $f_{\rm ISCO}=c^3/(6^{3/2}\pi Gm)$. The 3PN term depends on the unknown parameter $\hat\theta=\theta-\frac{7}{3}\lambda$ (we have $\hat\theta=\theta+1987/1320$ using the value of λ following from $\omega_{\rm static}=0$).

	$2\times1.4M_\odot$	$10M_\odot+1.4M_\odot$	$2\times10M_\odot$
Newtonian	16031	3576	602
1PN	441	213	59
1.5PN	−211	−181	−51
2PN	9.9	9.8	4.1
2.5PN	−12.2	−20.4	−7.5
3PN	$2.5+0.5\,\hat\theta$	$2.2+0.4\,\hat\theta$	$2.1+0.4\,\hat\theta$
3.5PN	−1.0	−1.9	−0.9

We shall now deduce the laws of variation of the frequency and phase using a balance equation as a fundamental tenet. Namely, we postulate that

$$dE/dt=-\mathcal{L}, \tag{10}$$

where the binary's gravitational binding energy E is given by Eq. (5), and where the total gravitational-radiation luminosity $\mathcal{L}$ is the one obtained in Eq. (9). For justification of the validity of the energy balance equation (10) in post-Newtonian approximations, for either point-particle binaries or extended weakly self-gravitating fluids, see Refs. [21], [22]. Using the previous formulas for E and $\mathcal{L}$, we transform Eq. (10) into an ordinary differential equation for ω or, rather, the parameter x. For convenience we adopt a new (dimensionless) time variable defined by

$$\tau=\nu c^3(t_c-t)/(5Gm), \tag{11}$$

where t_c denotes the instant of coalescence, at which the frequency tends formally to infinity (evidently, the approximation breaks down well before this point). The solution of the latter differential equation reads

$$x=\frac{1}{4}\tau^{-1/4}\Big(1+\Big(\frac{743}{4032}+\frac{11}{48}\nu\Big)\tau^{-1/4}-\frac{1}{5}\pi\tau^{-3/8}+\Big(\frac{19583}{254016}+\frac{24401}{193536}\nu+\frac{31}{288}\nu^2\Big)\tau^{-1/2}+\Big(-\frac{11891}{53760}+\frac{29}{1920}\nu\Big)\pi\tau^{-5/8}$$
$$+\Big\{-\frac{10052469856691}{6008596070400}+\frac{1}{6}\pi^2+\frac{107}{420}C-\frac{107}{3360}\ln(\tau/256)+\Big[\frac{15335597827}{3901685760}-\frac{451}{3072}\pi^2-\frac{77}{72}\lambda+\frac{11}{24}\theta\Big]\nu$$
$$-\frac{15211}{442368}\nu^2+\frac{25565}{331776}\nu^3\Big\}\tau^{-3/4}+\Big(-\frac{113868647}{433520640}-\frac{141389}{483840}\nu+\frac{275201}{3870720}\nu^2\Big)\pi\tau^{-7/8}\Big). \tag{12}$$

Next we compute the binary's instantaneous phase, defined as the angle ϕ, oriented in the sense of the motion, between the separation of the two bodies and, say, the direction of the ascending node of the orbit within the plane of the sky. We have $d\phi/dt=\omega$, which translates, in our notation, into $d\phi/d\tau=(-5/\nu)x^{3/2}$, and we can immediately integrate with the result

$$\phi=-\nu^{-1}\Big(\tau^{5/8}+\Big(\frac{3715}{8064}+\frac{55}{96}\nu\Big)\tau^{3/8}-\frac{3}{4}\pi\tau^{1/4}+\Big(\frac{9275495}{14450688}+\frac{284875}{258048}\nu+\frac{1855}{2048}\nu^2\Big)\tau^{1/8}+\Big(-\frac{38645}{172032}-\frac{15}{2048}\nu\Big)\pi\ln(\tau/\tau_0)$$
$$+\Big\{\frac{831032450749357}{57682522275840}-\frac{53}{40}\pi^2-\frac{107}{56}C+\frac{107}{448}\ln(\tau/256)+\Big[-\frac{123292747421}{4161798144}+\frac{2255}{2048}\pi^2+\frac{385}{48}\lambda-\frac{55}{16}\theta\Big]\nu$$
$$+\frac{154565}{1835008}\nu^2-\frac{1179625}{1769472}\nu^3\Big\}\tau^{-1/8}+\Big(\frac{188516689}{173408256}+\frac{140495}{114688}\nu-\frac{122659}{516096}\nu^2\Big)\pi\tau^{-1/4}\Big). \tag{13}$$

The constant τ_0 is related to a constant phase that is simply fixed by the initial conditions when the frequency of the wave enters the detector's bandwidth. Finally it can be useful to dispose of the expression of the phase in terms of the frequency x. For this we have

$$\phi=-(1/32)\nu^{-1}\Big\{x^{-5/2}+\Big(\frac{3715}{1008}+\frac{55}{12}\nu\Big)x^{-3/2}-10\pi x^{-1}+\Big(\frac{15293365}{1016064}+\frac{27145}{1008}\nu+\frac{3085}{144}\nu^2\Big)x^{-1/2}+\Big(\frac{38645}{1344}+\frac{15}{16}\nu\Big)\pi\ln(x/x_0)$$
$$+\Big(\frac{12348611926451}{18776862720}-\frac{160}{3}\pi^2-\frac{1712}{21}C-\frac{856}{21}\ln(16x)+\Big[-\frac{15335597827}{12192768}+\frac{2255}{48}\pi^2+\frac{3080}{9}\lambda-\frac{440}{3}\theta\Big]\nu$$
$$+\frac{76055}{6912}\nu^2-\frac{127825}{5184}\nu^3\Big)x^{1/2}+\Big(\frac{77096675}{2032128}+\frac{1014115}{24192}\nu-\frac{36865}{3024}\nu^2\Big)\pi x\Big\}, \tag{14}$$

where x_0 is determined by the initial conditions (like τ_0). As a rough estimate of the relative importance of each of the various post-Newtonian terms for LIGO/VIRGO-type detectors, we give in Table I their contributions to the accumulated number $\mathcal{N}$ of gravitational-wave cycles (see also Table I in Ref. [17] for the contributions of spin-orbit and spin-spin effects). The result for $\mathcal{N}_{3PN}$ is given as a function of the combination of parameters $\hat{\theta}=\theta-\frac{7}{3}\lambda$ that enters Eq. (14). As we can see, *if* $\hat{\theta}$ is of the order of unity, we reach with the 3PN or 3.5PN approximation an acceptable level of, say, a few cycles, which roughly corresponds to the demand which was made by data analyst in the case of neutron-star binaries [1,2]. Indeed, the above estimation suggests that the neglected 4PN terms will yield some systematic errors that are, at most, of the same order of magnitude, i.e., a few cycles, and perhaps much less. However, this conclusion is quite sensitive to the exact value of $\hat{\theta}$. *If* $\hat{\theta}$ is of the order of 10, we find that the 3PN term is nearly as important numerically as the previous 2PN approximation. Finally, in order to define the theoretical template of the compact binary inspiral, one should insert the previous 3.5PN-accurate expressions of the frequency and phase into the two polarization waveforms h_+ and $h_\times$. A standard practice is to neglect in h_+ and $h_\times$ all the harmonics but the dominant one f at twice the orbital frequency (i.e., the so-called restricted post-Newtonian approximation), but it is better to define the template by means of the 2PN-accurate polarization waveforms calculated in Ref. [23].

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